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A Structural Differencing Approach

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## THE TAXABLE INCOME ELASTICITY: A STRUCTURAL DIFFERENCING APPROACH

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# The Taxable Income Elasticity: A Structural Differencing Approach<sup>\*</sup>

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**Abstract:** We extend a standard taxable income model with its typical functional-form assumptions to account for nonlinear budget sets. We propose a new method to estimate a taxable income elasticity that is more policy relevant than the typically estimated elasticity based on linearized budget sets. Using U.S. data from the NBER tax panel for 1979-1990 and differencing methods, we estimate an elasticity of 0.75 for taxable income and 0.20 for broad income. These estimates are higher than those obtained by specifications based on linearization. Our approach offers a new way to address the problem of endogenous observed marginal tax rates.

**Keywords:** taxable income, nonlinear budget sets, panel data

**JEL classification:** D11, H24, J22

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# 1. Introduction

The responsiveness of taxable income to tax rate changes is widely recognized as an important research question in public finance. Following the work of Feldstein (1995, 1998), a large body of literature has emerged regarding estimation of the elasticity of taxable income with respect to the marginal net-of-tax rate at the observed taxable income.<sup>1</sup> The standard method adopted in much of the previous literature is to linearize the budget set at the observed taxable income and regress the change in the log of taxable income on the change in the log of observed marginal net-of-tax rate, using tax reforms as exogenous sources of variation in tax rates. The main econometric difficulty is that budget sets are nonlinear and an individual's observed marginal tax rate is mechanically a function of her income and, therefore, the change in marginal net-of-tax rate is endogenous to the change in taxable income.

Since the influential work of Gruber and Saez (2002), the taxable income literature has primarily addressed the endogeneity of the change in the log of observed marginal net-of-tax rate by instrumenting it with the change in the log of predicted marginal net-of-tax rate keeping real taxable income fixed at the base-year level. However, this instrumental variable remains endogenous, if income is mean reverting or if the income distribution is widening due to factors unrelated to tax rate changes. The literature has addressed these concerns by controlling for base-year income (Auten and Carroll, 1999; Saez, 2003; Gruber and Saez, 2002). Weber (2014), however, recently demonstrated that including controls for base-year income does not ensure instrument validity. She showed that even alternative income-based instrumental variables proposed in the literature (e.g., Carroll, 1998; Kopczuk, 2005; Blomquist and Selin, 2010) have similar drawbacks. Instead, she argued that instrumental variables based on income lagged multiple years are potentially valid.

In addition to concerns related to instrument validity, an important simplification in the previous literature is the implicit assumption that behavior under linearized budget sets closely approximates that when budget sets are nonlinear. While optimal income choices under both settings are the same if budget sets are convex, the estimated impact of tax changes can significantly differ if individuals change tax brackets. While the search for a valid instrumental variable to overcome the endogeneity of the observed marginal tax rate has been the focal point of the literature, estimation of the impact of tax policy leading to complicated budget set changes has received far less attention.

As a backdrop for a more concrete discussion of our contributions, we start by noting that in the standard empirical double-log specification of the taxable income function, the estimated elasticity with respect to the marginal net-of-tax rate corresponds to the elasticity of substitution parameter in a quasi-linear utility function when individuals face linear budget sets. In the presence of nonlinear budget sets, however, this elasticity of substitution (henceforth linear elasticity) only imperfectly captures the responsiveness of taxable income to tax rate changes. Blomquist et al. (2011, 2014) showed that a policy-specific taxable income elasticity can be defined with respect to changes in the entire nonlinear budget set by rotating the budget frontier. More generally, nonlinear budget sets could be characterized by several policy-relevant parameters not endogenous to income unlike the observed marginal

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<sup>1</sup> The net-of-tax rate is one minus the tax rate. See Saez et al. (2012) for a review of the literature.

net-of-tax rate. Different elasticity concepts could be defined with respect to these parameters (henceforth nonlinear elasticities). Building on their work, we define a nonlinear consumption net-of-tax elasticity corresponding to changing a flat consumption tax, the most natural analog of the linear elasticity. We show that this nonlinear elasticity is higher than the linear elasticity.

We then investigate the standard approach to estimating the elasticity with respect to the observed marginal net-of-tax rate for linearized budget sets when budget sets are nonlinear. We show that the estimated elasticity is a downward-biased estimate of the linear elasticity if there is preference heterogeneity, but that a valid instrumental variable can help overcome this bias. We argue, however, that the structural (instrumental-variable) estimate of the linear elasticity cannot be used to consistently simulate policy effects of tax changes, while the reduced-form estimate of taxable income on the instrumental variable yields a policy-relevant nonlinear elasticity.

Our main contribution is to extend the benchmark model with quasi-linear utility to incorporate nonlinear budget sets. We derive the taxable income function that depends on a weighted average of the marginal net-of-tax rates at every income level with the probability density function of unconditional income as weights. Since our specification accounts for the entire budget set, we circumvent the issues related to the observed marginal net-of-tax rate being endogenous to taxable income, such as mean reversion and widening income distribution, plaguing specifications based on linearized budget sets. A key advantage of our approach is that while it accounts for nonlinear budget sets, it can be estimated using panel data with differencing methods, much like the previous literature. The estimated (weighted) average marginal net-of-tax rate elasticity also has a direct policy-relevant interpretation as it could be interpreted as the nonlinear consumption net-of-tax elasticity.

From the perspective of our taxable income function, specifications based on linearization that include just the observed marginal net-of-tax rate and no other marginal net-of-tax rates are valid only when budget sets are linear. In addition to the previously known sources of endogeneity of the observed marginal net-of-tax rate related to mean reversion and widening income distribution, the omission of the remaining marginal net-of-tax rates is a previously unrecognized source of endogeneity. This is because observed and remaining marginal net-of-tax rates are very likely correlated. A valid instrumental variable for consistent estimation of the linear elasticity is significantly more demanding. The instrumental variable not only must be correlated with the observed marginal net-of-tax-rate, but also should be orthogonal to the remaining marginal net-of-tax-rates, to overcome the omitted-variable bias.

We also present an empirical application of our methodology on U.S. data from the NBER tax panel for 1979-1990 exploiting exogenous variation in tax reforms. This data set has been used several times previously for estimating the linear elasticity (e.g., Gruber and Saez, 2002).

We find an estimated nonlinear elasticity with respect to the average marginal net-of-tax rate of 0.75 for taxable income and 0.20 for broad income. The estimated elasticities are statistically significant at the one percent level. In comparison, the standard instrumental variable linearization method yields a reduced-form estimate of the nonlinear elasticity with respect to the marginal net-of-tax rate at the base-year income of 0.27 for taxable income and

0.13 for broad income. Accounting for nonlinear budget sets therefore increases the estimated elasticity. Unlike the standard method, our estimated elasticity with respect to the average marginal net-of-tax rate is insensitive to controlling for base-year income, indicating that these estimates are unaffected by bias due to mean reversion or widening income distribution. This result supports the view that budget set variation arising from tax reforms is exogenous when characterizing such variation in a manner that appropriately accounts for nonlinear budget sets.

We are not the first to account for nonlinear budget sets. Such methods have a well-established tradition in the labor supply literature that uses hours of work as the outcome variable. Burtless and Hausman (1978) and Hausman (1985) proposed a method in which estimation is carried out using maximum likelihood. Dagsvik (1994) and Hoynes (1996), among others, proposed an alternative method that discretizes the outcome variable and focuses on estimating parameters of the utility function. Blomquist and Newey (2002) developed a method allowing nonparametric estimation with least squares. Ongoing work by Blomquist et al. (2011, 2014) extends this method to estimation of taxable income. This paper differs from this literature by using the same arguably strong functional-form assumptions regarding the utility function and the probability density function of income (that we show was used implicitly before) as in the previous taxable income literature that linearized budget sets. By making these assumptions, we can provide a simple, transparent, and intuitive method that is amenable to use of panel data and differencing methods, and we can identify the bias due to linearization (and isolate it from the bias due to functional-form for instance).

The rest of the paper is organized as follows. The next section provides the theoretical analysis. Section 3 describes the empirical specifications and presents the data. Section 4 reports the elasticity estimates. The last section concludes.

## 2. Theoretical model

### 2.1 Model with linear budget sets

We start with a theoretical framework where the individual chooses  $(c, y)$  to maximize utility  $u(c, y)$  subject to a budget constraint  $c(y)$ , where  $c$  is consumption that equals the net after-tax income, and  $y$  is taxable income. The budget set is the area below the budget constraint.

Assuming a quasi-linear utility function will yield the double-log specification of the taxable income function estimated in the previous literature. With linear budget sets, the decision problem is:

$$\max_{c,y} u = -\frac{\beta_l^i \exp\left(-\frac{\alpha_l^i}{\beta_l^i}\right)}{\beta_l^i + 1} y^{\frac{\beta_l^i + 1}{\beta_l^i}} + c \quad \text{s.t.} \quad (1)$$

$$c = \tau y + c_0. \quad (2)$$

Superscript  $i$  indexes individuals and subscript  $l$  denotes parameters estimated when budget sets are linear.  $\alpha_l^i$  and  $\beta_l^i$  are both parameters of the utility function.  $\beta_l^i$  is the parameter representing the elasticity of substitution with respect to taxable income.  $\tau$  is the slope of the

budget frontier (i.e., consumption derivative with respect to taxable income  $c_y$ ) and equals the marginal net-of-tax rate.  $c_0$  is the intercept that equals net income from remaining residual sources and is assumed to be exogenous.

The optimal taxable income choice  $y^*$  for an interior solution is given by the first-order condition. Solving for  $y^*$  yields:

$$\ln y^* = \alpha_l^i + \beta_l^i \ln \tau. \quad (3)$$

With quasi-linear utility the optimal choice does not depend on the intercept and the income effect is, therefore, zero. Note that  $\beta_l^i = d \ln y^* / d \ln \tau$  is the elasticity of taxable income with respect to the marginal net-of-tax rate in linear budget sets (henceforth linear taxable income elasticity).

We now generalize the model in two ways. First, in order to capture differences in taste for work, we introduce two preference heterogeneity errors,  $e_1$  and  $e_2$ , and let  $\alpha_l$  and  $\beta_l$  be population-average parameters. Second, because individuals may not be able to precisely fine-tune their taxable income due to hours of work restrictions or availability of a limited set of taxable income options, we allow for optimization and measurement errors,  $e_3$ .<sup>2</sup> The error terms enter according to:

$$\alpha_l^i = e_1 \alpha_l, \quad (4)$$

$$\beta_l^i = e_2 \beta_l, \quad (5)$$

$$y = e_3 y^*. \quad (6)$$

We assume that the sources of error vary between individuals, have a mean of one, and are statistically independent conditional on the budget set. We omit the  $i$  index on them for notational simplicity.

The expected taxable income can then be written as:

$$E(\ln y) = \alpha_l + \beta_l \ln \tau. \quad (7)$$

Eq. (7) shows that accounting for preference heterogeneity and optimization errors according to Eqs. (4) to (6) still leads to a double-log specification. Therefore, the models estimated in the previous literature are generally consistent with the presence of such complications and could be used to estimate the population-average elasticity  $\beta_l = E(d \ln y^* / d \ln \tau) = E(d \ln y / d \ln \tau)$ .

Letting  $z$  be a component of the preference heterogeneity terms  $e_1$  and  $e_2$  that can be accounted for, we only require independence of the error terms conditional on  $z$ . In the empirical differencing estimation setting that we later apply, individual-specific heterogeneity is differenced away, relaxing the independence assumption on the error terms considerably.

The taxable income functions in Eqs. (3) and (7) are obtained analytically by solving for  $y^*$  from the first-order condition that depends on a marginal tax rate that is constant across income levels and therefore not endogenous to income. In nonlinear budget sets, there are many marginal tax rates and the *observed* marginal tax rate is endogenous to income. We present an iterative procedure to derive the taxable income function that is useful for handling this complication later on when we extend the model to account for nonlinear budget sets. The

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<sup>2</sup>  $e_3$  is the product of optimization and measurement errors.

procedure involves increasing  $y$  in steps and checking at each step whether the first-order condition holds using the relevant marginal tax rate (which is constant in linear budget sets). Using  $j$  to denote the running variable index, the taxable income function can then be expressed as:

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j f(\ln y_j | \ln \tau, e_1, e_2) f(e_1) f(e_2) d(\ln y_j) de_1 de_2, \quad (8)$$

$$f(\ln y_j | \ln \tau, e_1, e_2) = 1(\ln y_j = e_1 \alpha_l + e_2 \beta_l \ln \tau). \quad (9)$$

where  $f(\ln y_j | \ln \tau, e_1, e_2)$  is a conditional probability density function, and  $1(\cdot)$  is an indicator function. Note that, because of the independence assumption on the error terms, we could make use of  $f(\ln y_j, e_1, e_2 | \ln \tau) = f(\ln y_j | \ln \tau, e_1, e_2) f(e_1) f(e_2)$  in deriving Eq. (8).

For Eq. (8) to equal the double-log specification in Eq. (7), we require:

$$\int \ln y_j f(\ln y_j | \ln \tau, e_1, e_2) d(\ln y_j) = \int (e_1 \alpha_l + e_2 \beta_l \ln \tau) f(\ln y_j) d(\ln y_j). \quad (10)$$

This clarifies the exact implicit functional-form assumption on the conditional probability density function of income needed to obtain the double-log specification of the taxable income function applicable to linear budget sets. We show in Subsection 2.4 the implications of making a similar functional-form assumption when budget sets are nonlinear.

## 2.2 Nonlinear budget set elasticity

Real-world tax and transfer systems, being progressive, lead to piecewise-linear budget sets. A work-around is to linearize the budget set at the observed taxable income to obtain the observed marginal net-of-tax rate and estimate the linear taxable income elasticity assuming a linear budget set. A rationale for this procedure is that the optimal choice on the linearized budget set is the same as the optimal choice on the nonlinear budget set if preferences and budget set are convex (Hausman, 1985; Mofitt, 1990). Varying the observed marginal net-of-tax rate of the linearized and nonlinear budget sets, therefore, has the same effect on behavior as long as the individual stays within the same tax bracket after the variation.

Assume for simplicity that the nonlinear budget frontier is differentiable.<sup>3</sup> The first-order condition in Eq. (3), from the optimization problem in Eqs. (1) and (2) with the linearized budget set, continues to be both sufficient and necessary for an optimum. The complication is that when the budget set is nonlinear, the marginal net-of-tax rate is a function of taxable income:

$$\ln \tau(\ln y) = \ln c_y(\ln y). \quad (11)^4$$

The linearized marginal net-of-tax rate needed to obtain the taxable income in Eq. (3) is therefore, not constant; because it depends on taxable income, it is endogenous. In this setting,

<sup>3</sup> Most budget sets are piece-wise linear in which case the budget frontier is not differentiable. Individuals may then want to locate at kinks which are the main complications with nonlinear budget sets. The linearization approach ignores such complications. Assuming that the nonlinear budget frontier is smooth simplifies the analysis and still illustrates the essential complications with nonlinear budget sets.

<sup>4</sup> The nonlinear budget constraint comparable to Eq. (2) is  $c = c(y)$ . We express the budget set in terms of a marginal net-of-tax rate function and take logs of the variables here because this is the function relevant for elasticities.

the chosen taxable income,  $y^*$ , and marginal net-of-tax rate,  $\tau^*$ , are the solutions to the equation system given by Eqs. (3) and (11).<sup>5</sup>

An important question is whether the elasticity of substitution in the utility function,  $\beta_l^i$ , remains an interesting parameter when budget sets are nonlinear. Under linear budget sets,  $\beta_l^i$  conveniently represents the elasticity of taxable income with respect to a unique marginal net-of-tax rate that is a budget-set parameter that can be directly controlled by tax policy. In nonlinear budget sets, both chosen taxable income and marginal tax rate are outcome variables that cannot be directly controlled by tax policy. Instead, there are many policy-relevant marginal tax rates, and therefore many ways to vary the marginal net-of-tax rate function. Policy-relevant nonlinear elasticities may be defined with respect to any subset of these rates.

We note that the linear taxable income elasticity corresponds to a tax schedule change where marginal net-of-tax rates are changed *proportionally* at every income level. Such an overall tax schedule change that proportionally rotates the budget frontier is possible also when the tax schedule is nonlinear, as noted by Blundell and Shephard (2012). Such rotation can be thought of as capturing the change in the budget set due to the change in a flat consumption tax and is best describe as a nonlinear consumption net-of-tax elasticity and can be expressed as follows:

$$\beta_{nl}^i = \frac{d \ln y^*}{d \ln \tau(\cdot)}, \quad \beta_{nl} = E(\beta_{nl}^i). \quad (12)^6$$

This concept of nonlinear elasticity is the most natural analog of the linear elasticity. Note that, it collapses to the linear elasticity when the marginal-net-of-tax-rate is constant, i.e., when  $\ln \tau(\cdot) = \ln \tau$ .

The consumption net-of-tax elasticity is similar in spirit to the nonlinear elasticity defined by Blomquist et al. (2011, 2014) that rotates the budget frontier upwards in absolute terms.<sup>7</sup> Besides providing a policy-relevant overall measure of tax rate effects, the consumption net-of-tax elasticity also corresponds to a parameter in the taxable income function with nonlinear budget sets, as we will show in Subsection 2.4. For now, we note that we could directly estimate the consumption net-of-tax elasticity by regressing (the change in) taxable income on (the change in) the consumption net-of-tax rate, if the (change in) consumption tax is exogenous.

We now investigate how the nonlinear elasticity  $\beta_{nl}$  relates to the linear elasticity  $\beta_l$  when budget sets are nonlinear. Plugging Eq. (11) into Eq. (3) and differentiating with respect to  $\ln \tau(\cdot)$  assuming that  $\ln \tau(\cdot)$  is differentiable, and solving for  $\beta_{nl}^i$  yields:

<sup>5</sup> The nonlinear budget set model in Burtless and Hasusman (1978) and Hausman (1985) is equivalent to our Eqs. (3) and (11) for the case with piecewise-linear budget sets.

<sup>6</sup> More generally, let  $\ln \tau = \ln \tau(\ln y; \theta)$  where  $\theta$  is a budget set parameter that can be controlled by tax policy. A nonlinear elasticity for tax policy  $d\theta$  could be defined as  $d \ln y / d\theta$ . The consumption net-of-tax elasticity results when the policy parameter enters according to  $\ln \tau(\cdot) = \ln \tau(\ln y) + \theta$ .

<sup>7</sup> Their absolute rotation corresponds to changing a linear local tax. In linear budget sets, absolute or relative rotations are identical up to a scaling factor. In nonlinear budget sets, this is not the case. An absolute rotation has theoretically attractive features by keeping the intercept income effects of any linearized budget sets fixed. However, generalizing a *constant* linear elasticity leads to a nonlinear elasticity that corresponds to a relative rotation.

$$\beta_{nl}^i = \frac{\beta_l^i}{1 - \beta_l^i \ln \tau_{\ln y}} \leq \beta_l^i. \quad (13)^8$$

Our nonlinear elasticity is lower than the linear elasticity when the tax system is progressive since  $\ln \tau_{\ln y} < 0$  in this case. The intuition is that increasing marginal net-of-tax rates has a direct positive effect on income when the budget set is linearized. However, the individual may enter new tax brackets with higher marginal tax rates (or lower marginal net-of-tax rates), leading to a subsequent counteracting negative effect on income. The difference between the linear and nonlinear elasticity is negatively related to the linear elasticity and positively related to the progressivity of the tax system.<sup>9</sup>

### 2.3 The linearization approach

We now investigate the standard approach to estimating the elasticity with respect to the observed marginal net-of-tax rate for linearized budget sets when budget sets are nonlinear. Without loss of generality, let us consider for example where all the variation in taxable income is generated by an exogenous variation in a consumption tax. The association between the chosen taxable income and marginal-net-of tax rate can then be expressed as follows:

$$\frac{d \ln y^*}{d \ln \tau^*} \Big|_{\ln \tau(\cdot) \text{ varies}} = \frac{\ln y^*_{\ln \tau(\cdot)}}{\ln \tau^*_{\ln \tau(\cdot)}} = \frac{\frac{\beta_l^i}{1 - \beta_l^i \ln \tau_{\ln y}}}{\frac{1}{1 - \beta_l^i \ln \tau_{\ln y}}} = \beta_l^i. \quad (14)^{10}$$

The marginal net-of-tax elasticity in Eq. (3) using the observed marginal net-of-tax rate and linearized budget sets, therefore, equals the elasticity of substitution in the utility function when budget sets are linear. Intuitively, as the individual enters tax brackets with higher tax rates, there is a negative effect on the chosen marginal net-of-tax rate (denominator in Eq. (14)) which is of the same size as the counteracting effect on the chosen taxable income. The two effects cancel out. It can be shown that any other type of exogenous budget set variation can be used to estimate the linear elasticity.<sup>11</sup>

In addition to variation in budget sets, the association between taxable income and observed marginal net-of-tax rate may also be affected by preference heterogeneity. Allowing for preference heterogeneity according to Eq. (5), the association due to such heterogeneity is:

<sup>8</sup> Eqs. (3) and (11) yield  $\ln y^* = \alpha_l^i + \beta_l^i \ln \tau (\ln y^*)$ . Differentiating with respect to  $\ln \tau (\cdot)$  yields  $\beta_{nl}^i = \beta_l^i + \beta_l^i \ln \tau_{\ln y} \beta_{nl}^i$ . Rearranging yields Eq. (13).

<sup>9</sup> Blomquist et al. (2011, 2014) also showed an example where their nonlinear elasticity is lower than the linear elasticity when the budget frontier is piecewise linear and quasi-concave. They also conjectured that welfare effects are more closely related to a nonlinear elasticity than the linear elasticity.

<sup>10</sup> Writing the relationship between  $\ln y^*$  and  $\ln \tau^*$  as  $\ln y^* = \gamma \ln \tau^*$ , we see that  $d \ln y^* / d \ln \tau^* = \gamma$  is the regression coefficient. The numerator in Eq. (14) is given by Eq. (13). Eqs. (3) and (11) yield  $\ln \tau^* = \ln \tau (\alpha_l^i + \beta_l^i \ln \tau^*)$ . Differentiating with respect to  $\ln \tau (\cdot)$  yields  $\ln \tau^*_{\ln \tau(\cdot)} = 1 + \ln \tau_{\ln y} \beta_l^i \ln \tau^*_{\ln \tau(\cdot)}$ . Rearranging yields the denominator in Eq. (14).

<sup>11</sup> Following footnote 6, we could show that  $d \ln y^* / d \ln \tau^* |_{\theta \text{ varies}} = \ln y^*_{\theta} / \ln \tau^*_{\theta} = \beta_l^i$ .

$$\frac{d \ln y^*}{d \ln \tau^*} \Big|_{e_2 \text{ varies}} = \frac{\ln y^* e_2}{\ln \tau^* e_2} = \frac{\frac{\beta_l \ln \tau}{1 - e_2 \beta_l \ln \tau_{\ln y}}}{\frac{\ln \tau_{\ln y} \beta_l \ln \tau}{1 - e_2 \beta_l \ln \tau_{\ln y}}} = \frac{1}{\ln \tau_{\ln y}} < 0. \quad (15)^{12}$$

The intuition is that, due to the progressivity of the tax system, individuals with a high taste for work choose higher incomes,  $y^*$ , and lower marginal net-of-tax rates at these income levels,  $\tau^*$ , leading to a negative association. The association between these variables observed in the data typically reflects a mixture of the variations in preferences and budget sets, whereas we are interested in sorting out the effect of preferences to isolate the effect of budget sets.

An empirical approach to isolate and estimate the effect of taxes on taxable income would be to use an instrumental variable that depends only on exogenous variation in the tax schedule. For a consumption-tax based instrumental variable, Eq. (14) shows that the direct reduced-form effect of the instrumental variable on taxable income in the numerator provides the nonlinear consumption net-of-tax elasticity. Reduced-form estimates of nonlinear elasticities are, however, specific to the type of tax-schedule variation the instrumental variable is based on. Eq. (14) also shows that the denominator is the first-stage effect and the ratio between the reduced-form and first-stage effects yields the structural estimate of the linear elasticity. Unlike the reduced-form estimates of the nonlinear elasticity, the structural instrumental-variable estimate of the linear elasticity is biased if the instrumental variable is weak.

Introducing optimization or measurement errors causes additional complications. Not only would the observed taxable income differ from the optimal taxable income choice, but also the observed marginal net-of-tax rate, used to proxy the rate at the optimal choice, would contain an error. For the estimation of a nonlinear elasticity, the error only enters the dependent variable and, therefore, does not lead to bias. In case of the linear elasticity, however, the error enters an independent variable and would, without use of an instrumental variable uncorrelated with the errors, induce a bias (e.g., an attenuation bias if the error is normally distributed). In the next subsection, we discuss an additional problem that arises when trying to estimate the linear elasticity consistently using the linearization approach.

Even if the linear elasticity could be consistently estimated, it is less useful for simulating taxable income responses to tax rate changes. With individual heterogeneity, we can only estimate the population-average linear taxable income elasticity in Eq. (7) and not the individual-specific elasticities in Eq. (3) needed for simulations. To evaluate a consumption tax reform, we are, e.g., interested in  $\beta_{nl} = E(\beta_l^i / (1 - \beta_l^i \ln \tau_{\ln y}^i)) \neq E(\beta_l / (1 - \beta_l \ln \tau_{\ln y}^i))$ .

## 2.4 Taxable income function with nonlinear budget sets

In this subsection, we extend the outlined benchmark model with the quasi-linear utility function in Eq. (1) to incorporate nonlinear budget sets. Rather than linearizing budget sets, we derive a structural taxable income function that accounts for nonlinear budget sets. The

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<sup>12</sup> The derivation follows the same procedure as outlined in Footnotes 8 and 10.

only additional assumption needed is a modification of the functional-form assumption on the probability density function of income in Eq. (10). Replacing the linear budget constraint in Eq. (2) by the nonlinear budget constraint in Eq. (11) and allowing for preference heterogeneity and optimization errors according to Eqs (4) to (6), the first-order condition and the optimal taxable income can now be written as:

$$\ln y^* = e_1 \alpha_l + e_2 \beta_l \ln \tau^*. \quad (16)$$

Importantly,  $\tau^*$  is endogenous, unlike  $\tau$  is Eq. (3).<sup>13</sup>

We can derive the taxable income function using the theoretical iterative procedure described previously for linear budget sets in Subsection 2.1, which consists of increasing  $y$  in steps and at each step checking whether the first-order condition underlying Eq. (16) holds. This is feasible because at each fixed  $y_j$ ,  $\tau_j$  is exogenous, if the budget set is exogenous. Given the independence assumptions on the error terms, we then obtain:

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j f(\ln y_j | \ln \tau_j, e_1, e_2) f(e_1) f(e_2) d(\ln y_j) de_1 de_2, \quad (17)$$

$$f(\ln y_j | \ln \tau_j, e_1, e_2) = 1(\ln y_j = e_1 \alpha_l + e_2 \beta_l \ln \tau_j), \quad (18)$$

which is a straightforward generalization of the expression for linear budget sets in Eqs. (8) and (9). The only difference is that  $\tau_j$  varies with  $y_j$ . Making the functional-form assumption similar to Eq. (10) we get:

$$\int \ln y_j f(\ln y_j | \ln \tau_j, e_1, e_2) d(\ln y_j) = \int (e_1 \alpha_{nl} + e_2 \beta_{nl} \ln \tau_j) f(\ln y_j) d(\ln y_j), \quad (19)$$

which yields the taxable income function:

$$E(\ln y) = \alpha_{nl} + \beta_{nl} \int \ln \tau_j f(\ln y_j) d(\ln y_j). \quad (20)$$

This taxable income function depends on a weighted average of marginal net-of-tax rates on the budget frontier rather than the marginal net-of-tax rate at a single point. The weight is the probability density function of the unconditional income. For estimation, we could discretize the budget set and numerically integrate over points on the budget frontier, and we could approximate the probability density function by the sample probability distribution.<sup>14</sup> Since our specification depends on the entire budget set, we circumvent the issues related to the observed marginal net-of-tax rate being endogenous to income, such as mean reversion and widening income distribution, plaguing specifications based on linearized budget sets.

Note that, unlike linear budget sets that require just two parameters to characterize ( $\tau$  and  $c_0$ ), the taxable income function with nonlinear budget sets depend on the function  $\ln \tau(\cdot)$  that consists of a large number of parameters. Nevertheless, estimation remains tractable because the taxable income function is an integral over a two-dimensional term ( $\tau_j$  and  $j$ ).

<sup>13</sup> Kink points can be handled by modifying the first-order condition so that we require  $FOC > 0$  at  $y^* - \epsilon$  and  $FOC < 0$  at  $y^* + \epsilon$ . Corner solutions can be handled in a similar manner. In Eq. (16), we could replace  $\tau^* = \tau(y^*)$  by  $\tau^* = \tau(y = \lim_{\epsilon \rightarrow 0^+} y^* + \epsilon)$  or  $\tau^* = \tau(y = \lim_{\epsilon \rightarrow 0^-} y^* + \epsilon)$ . It can be shown in a more fullfledged nonlinear budget set model that the two specifications bound the true estimate, where the bounds decrease with the spread of the optimization errors. In practice, both specifications yield almost identical estimates.

<sup>14</sup> We set  $\tau_j = (c_{j+1} - c_j) / (y_{j+1} - y_j)$  in the numerical integration.

Compared with the linear budget set case (that depends only on  $\tau$ ), the dimensionality in the nonlinear case increases only by one. This is because the first-order conditions are the same at every point on the budget frontier and the optimality at each point depends only on the marginal net-of-tax rate at that point.

Note also that the estimated parameters,  $\alpha_{nl}$  and  $\beta_{nl}$  in Eqs. (19) and (20) are not equal to  $\alpha_l$  and  $\beta_l$  in Eqs. (16) and (18), unless budget sets are linear. The expected nonlinear budget set function nests the expected linear budget set function.  $\beta_{nl}$  is a nonlinear elasticity with respect to the (weighted) average marginal net-of-tax rate and could be used to simulate the effect of tax reforms on taxable income based on reform-induced changes in average marginal net-of-tax rates.  $\beta_{nl}$  could also be interpreted as the nonlinear consumption net-of-tax elasticity in Eq. (12) as a unit increase in the log of marginal net-of-tax rates at every income level results in a unit increase in the weighted average of the log of marginal net-of-tax rates.

In the presence of nonlinear budget sets, preference heterogeneity, and optimization errors, the specification of the taxable income function in Eq. (7) based on linearization – that includes just the observed marginal net-of tax rate – is essentially misspecified. Eq. (20) shows that marginal net-of-tax rates at income levels other than the observed level also affect taxable income. In addition to the previously known sources of endogeneity of the observed marginal net-of-tax rate related to mean reversion and widening income distribution, the omission of the remaining marginal net-of-tax rates is a previously unrecognized source of endogeneity. The reason is that observed and remaining marginal net-of-tax rates are very likely correlated. A valid instrumental variable for consistent estimation of the linear elasticity is significantly more demanding. The instrumental variable not only needs to be correlated with the observed marginal net-of-tax-rate, but also should be orthogonal to the remaining marginal net-of-tax-rates, to overcome the omitted-variable bias.<sup>15</sup>

If one wishes to obtain an estimate of the linear elasticity, the proper way would be to estimate the population-average nonlinear elasticity in Eq. (20) and then to derive and make use of an equivalent of Eq. (13) that accounts for optimization errors and that applies to population-average elasticities. By focusing on nonlinear elasticities in the next empirical sections, we avoid this complication.

### 3. Empirical estimation

#### 3.1 Regression specifications

It is possible to estimate the taxable income function in Eq. (20) using a single cross-section. However, the independence assumption of the budget set is unlikely to hold because budget sets typically correlate with demographic variables that may also affect taxable income. To

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<sup>15</sup> The omitted-variable problem is also present for the reduced-form estimate of taxable income on any instrumental variable representing an estimated nonlinear elasticity. Consider, e.g., the reduced-form estimate of taxable income on a first-dollar marginal net-of-tax rate instrumental variable. Such an estimate may not provide a consistent estimate of the taxable income response to a tax reform that varies the first-dollar marginal net-of-tax rate, if the first-dollar marginal net-of-tax rate correlates with remaining marginal net-of-tax rates. Inflating the reduced-form estimate by the first-stage estimate to obtain the structural estimate of the linear elasticity, however, additionally inflates the bias.

the extent that these variables are individual-specific, their effects can be differenced away if panel data is available. We could then estimate the basic specification:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl} \sum_j \Delta \ln \tau_{j,i,t} \Pr(\ln y_j) + \varepsilon_{i,t}, \quad (21)$$

where  $i$  index individuals,  $t$  index year,  $\Delta$  is the difference operator,  $\alpha_{i,t}$  is a constant, and  $\varepsilon_{i,t}$  is an idiosyncratic error term. In our application,  $j$  indexes 200 income intervals where the first 199 each covers 1,000 dollars, and the 200<sup>th</sup> covers the open interval above 199,000 dollars.  $\Pr(\ln y_j)$  is the observed probability of taxable income  $y$  being in interval  $j$ .  $\tau_{j,i,t}$  is the average marginal net-of-tax rate in  $j$ .

In our baseline specification, we use three-year differences. Obviously, identification requires at least one tax reform that differently affected individuals' budget sets. To the extent that these tax reforms lead to exogenous budget set changes, the nonlinear elasticity with respect to the (weighted) average marginal net-of-tax rate, which also could be interpreted as the consumption net-of-tax elasticity, can be consistently estimated. In our empirical application, we stack differences from eight different years and make use of multiple tax reforms. Because we have single filers (singles and couples that choose to file separately) and joint filers (couples), we also control for filing status.

Note that constant relative changes in taxable income over years and between individuals — due to, e.g., productivity growth — are accounted for by the intercept term in Eq. (21). To remove any potential correlation between the timing of the different tax reforms and differences in productivity growth between years, we also include year dummies.

In comparison, the standard method is to linearize the budget set at the observed taxable income and estimate the effect of the change in the log of observed marginal net-of-tax rate on the change in the log of taxable income. The endogenous change in observed marginal net-of-tax rate is instrumented by the change in predicted marginal net-of-tax rate at the base-year income level. Our specification in Eq. (21) is similar to the structural equation, but replaces the change in the log of observed marginal net-of-tax rate by the change in the log of weighted average marginal net-of-tax rate. We, thus, avoid the endogeneity of the observed marginal net-of-tax rate to income.

As discussed in Subsection 2.3, nonlinear elasticities correspond to reduced-form effects of instrumental variables on taxable income. Our nonlinear elasticity of the average marginal net-of-tax rate is therefore more comparable to the reduced-form estimate of the nonlinear elasticity with respect to the marginal net-of-tax rate at the base-year income than the structural estimate of the linear elasticity in the standard linearization specification. We therefore compare our estimates with estimates from the following standard reduced-form linearization specification:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl} \Delta \ln \tau_{y,i,t} + \varepsilon_{i,t}, \quad (22)$$

where the marginal net-of-tax rate  $\tau_{y,i,t}$  is evaluated at the base-year income level  $y$ . Comparing our specification that accounts for nonlinear budget sets in the estimation in Eq. (21), with this reduced-form specification, the difference is that we use changes in marginal net-of-tax rates averaged over income levels rather than at a single income level. The standard specification also usually includes year dummies. Any difference between estimated

elasticities from Eqs. (21) and (22) reflects the bias from not accounting for nonlinear budget sets *in the estimation*, although both provide elasticities of taxable income with respect to nonlinear budget sets.

An issue with the standard instrumental variable ( $\Delta \ln \tau_{y,i,t}$ ), based on the observed base-year income, is that it is not valid because observed base-year income may be directly correlated with taxable income. This is the case if there is a temporary component of taxable income causing mean reversion, in which case individuals with high income in one year tend to revert toward the mean in the next year. Another issue is heterogeneous productivity growth between individuals with different background characteristics who also have different income levels. Income inequality trends, such as a widening income distribution over time, are an example of such an issue.

The standard method to address both mean reversion and heterogeneous income growth is to control for either the log of base-year income or its spline. Because the instrumental variable and the spline both rely on variation in base-year income, identification of the marginal net-of-tax rate parameter could be problematic and could possibly depend heavily on functional-form assumptions. The availability of data spanning several reforms may help identification by providing variation in the change in marginal net-of-tax rate at the same income level for different base years. Of course, this only helps if the effect of base-year income is the same for different base years.

Weber (2014) investigated whether the standard instrumental variable and other related instrumental variables that are functions of taxable income are valid. She found that they cannot overcome the mean reversion problem, even when they include a base-year income control function. She showed, however, that income lagged several years could be used to construct instrumental variables that are valid in the limit, because the temporary component of taxable income diminishes over time. As for the issue of heterogeneous productivity growth, including an income control function is more promising, although again, base-year income is endogenous. Weber suggested using longer income lags for this issue as well.

Because our independent variable is not a function of income, there is no automatic correlation with factors that correlate with income in any year. To the extent that tax reforms are exogenous and not correlated with income, there is no need to include an income control function.

It is, of course, still possible that tax reforms may correlate with other groups-specific trends. For example, reforms may target groups with certain background characteristics, which may have different income trends. This could be addressed by including demographic control variables. We do not have such variables. Base-year income could, however, be a proxy for such heterogeneity and solve the issue to the extent that such controls correlate with income. We include a base-year income control function to check the sensitivity of our estimates and to assess the exogeneity of tax reforms assumption. We also include such control functions in our standard reduced-form linearization specification.

### **3.2 Data**

We use data from the NBER panel of tax returns over the 1979-1990 period, also known as the Continuous Work History File, which is also the data used by Gruber and Saez (2002) and

Weber (2014). The data contain detailed administrative information on tax and income variables but do not contain any demographic background variables. See Gruber and Saez's paper for a detailed description of the data set.

We construct the individual budget sets by computing marginal net-of-tax rates at different income levels using NBER-TAXSIM. We vary earnings and capital income in steps of 1000 USD (1990 price level) keeping income from other sources fixed. Deductions/itemizations that vary between individuals are accounted for in the construction of the budget sets.

We use two of the most common measures of taxable income used in the previous literature: actual taxable income (almost exactly as technically defined in the tax forms) and broad income. Broad income is an extensive definition of gross income and includes, among other things, wage income, interest income, dividends, and business income. We exclude capital gains, however. Taxable income is broad income minus a number of deductions. We use the definition in 1990 and include all adjustments that can be computed from the data for all sample years. Our definitions are consistently defined over the entire sample period and are identical to the ones used by Gruber and Saez (2002).

As discussed by Gruber and Saez (2002), one margin of behavioral responses to tax reforms is to shift income between different sources, e.g., between taxable income and other components of broad income. To the extent that composition of income is endogenous to the constructed budget sets, the taxable income estimates would be biased. The broad income estimates should, however, be less problematic in this regard. The taxable income estimates do, however, include the effects of tax avoidance, which is also a margin of interest for policy and welfare evaluations.<sup>16</sup>

Our sample selection criteria are similar to the ones in Gruber and Saez (2002). We drop filers that change filing status and observations with abnormally large (top and bottom one percent of the sample) changes in the weighted average marginal net-of-tax rate since these observations are more likely to be driven by variable construction errors. We also drop observations where we could not compute either taxable income or broad income. However, we do not truncate our data from below, unlike Gruber and Saez. We use the log of income plus one as the outcome variable to enable inclusion of observations that involve individuals with zero income. Sample statistics can be found in the Appendix.

## 4. Results

### 4.1 Taxable income

In Table 1, we report estimates of the nonlinear taxable income elasticities in Eqs. (21) and (22). In the first rows, we report the estimates of the change in the log of (weighted) average marginal net-of-tax rate as defined in Eq. (21). This is the specification that fully accounts for nonlinear budget sets. In the last rows, we report the estimates of the change in the log of predicted marginal net-of-tax rate at the base-year income level. This is the reduced-form

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<sup>16</sup> To properly investigate income shifting effects would require a model that allows for several choice variables. The dimensionality of the budget set would then be the number of choice variables plus one.

estimate of taxable income on the instrumental variable in the standard linearization method. In Column (1), no control variables are included. In subsequent columns, we successively add a dummy for filing status, year dummies, the log of base-year income, and a ten-piece spline in the log of base-year income.

Table 1. Estimates of nonlinear elasticities of taxable income

	(1)	(2)	(3)	(4)	(5)
Change log average marginal net-of-tax rate	0.559 <sup>**</sup> (0.056)	0.600 <sup>**</sup> (0.056)	0.756 <sup>**</sup> (0.059)	0.742 <sup>**</sup> (0.054)	0.748 <sup>**</sup> (0.052)
Change log predicted marginal net-of-tax rate	-0.146 <sup>**</sup> (0.025)	-0.110 <sup>**</sup> (0.025)	-0.082 <sup>**</sup> (0.026)	0.358 <sup>**</sup> (0.023)	0.269 <sup>**</sup> (0.023)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The change in the log of taxable income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. \*  $p < 0.05$ ; \*\*  $p < 0.01$ .

Our estimated elasticity of taxable income with respect to the average marginal net-of-tax rate that accounts for nonlinear budget sets in the estimation is 0.56 when no other covariates are included and around 0.75 after controlling for filing status and year effects. The estimated elasticities are all statistically significant at the one percent level, and they do not change much when we additionally control for base-year income, either in the form of the log of base-year income or a spline in the log of base-year income. The evidence, therefore, does not contradict that the budget set variation between individuals arising from tax reforms, as represented by the change in the log of average marginal net-of-tax rate, is exogenous. An elasticity of 0.75 implies, e.g., that a one percent increase in marginal net-of-tax rates at every income level, which could be carried out by decreasing the tax rate of a flat consumption tax, leads to an increase in taxable income of 0.75 percent.

In comparison, the estimated elasticity of taxable income with respect to the marginal net-of-tax rate at the base year income based on linearized budget sets (in the estimation) is negative in Column (1). This likely reflects a correlation that is at least partly caused by preference heterogeneity, as shown by Eq. (15). Controlling for base-year income turns the estimated elasticity positive, indicating that mean reversion and/or heterogeneous productivity growth induce a bias for the standard instrumental variable. Also, the exact estimated elasticity is sensitive to the functional form of the base-year income control. The specification with the spline corresponds to the reduced-form estimates of the standard method and produces an elasticity of 0.27. Again, all estimated elasticities are statistically significant at the one percent level. Comparing the two sets of estimated elasticities in Table 1 indicates that accounting for nonlinear budget sets increases the estimated elasticity by almost a factor of three.

Our linearized budget set estimates could be compared to those from Gruber and Saez's (2002) on the same data but a slightly different sample. Their estimated taxable income elasticity estimate of 0.40 from the specification that includes a spline could be compared to

our estimates in Column (5). There are three sources of difference between our and their estimated elasticities. First and most importantly, they estimate the structural equation and therefore obtain the linear elasticity. Second they weight their regression by taxable income. And finally, their sample differs slightly from ours. Their reduced-form estimate (based on our estimations since they did not report this estimate) would have been 0.21 and statistically significant at the one percent level, which is close to 0.27 that we obtain. This translates into a first-stage estimate around 0.53.<sup>17</sup>

Weber (2014) used instrumental variables based on longer income lags on the same data. She found a linear taxable income elasticity between 0.86 and 1.36 in most specifications. Because she used several instrumental variables, there are several first-stage and reduced-form estimates (one for each instrumental variable) and there is not a single reduced-form estimate for us to compare with. Rescaling her (structural) estimates by Gruber and Saez’s first-stage estimate (0.53 based on our estimations) results in a nonlinear elasticity between 0.46 and 0.72, which would still be smaller than our estimated elasticity.<sup>18</sup>

To illustrate our estimates and the influence of base-year income, we plot the change in the log of taxable income variable against the two different change in the log of marginal net-of-tax rate variables in Figure 1. We also plot these two budget set variables against the log of base-year income in Figure 2. We use both a local polynomial and a linear fit to the data in Figure 1, whereas we only use the local polynomial fit in Figure 2.

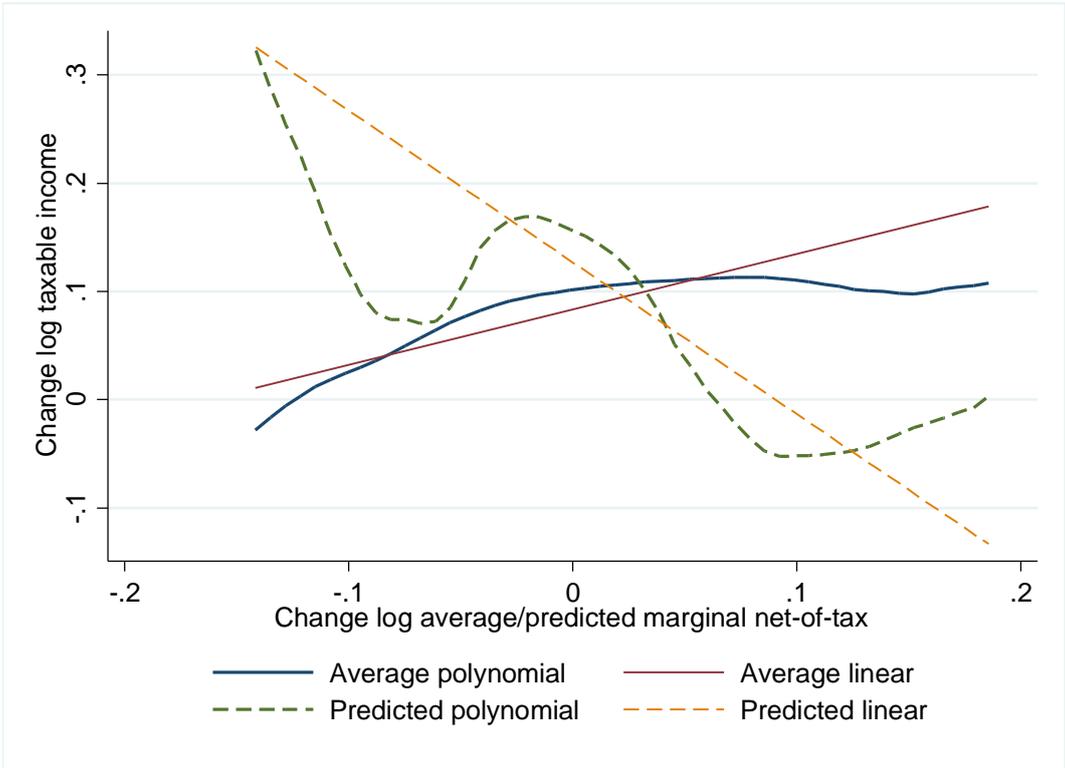


Figure 1. Graphs of the outcome variable against the marginal net-of-tax rate variables

<sup>17</sup> The structural estimate (0.40) equals the reduced-form estimate (0.21) divided by the first-stage estimate (0.53) according to Eq. (14).

<sup>18</sup> The computations are done in the same way as described in footnote 17.

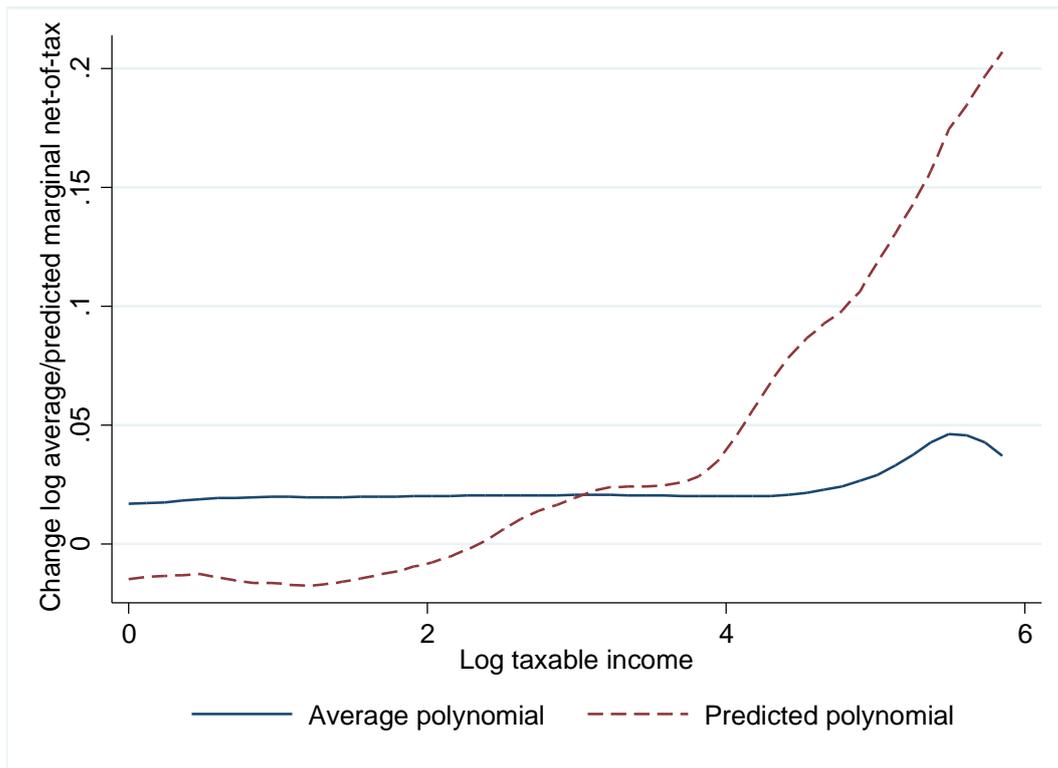


Figure 2. Graphs of the marginal the net-of-tax rate variables against base-year income

Figure 1 shows that the correlation between the outcome variable and the change in average marginal net-of-tax rate is positive, whereas the correlation between the outcome variable and the change in predicted marginal net-of-tax rate is negative. In Figure 2, base-year income is uncorrelated with the change in average marginal net-of-tax rate, supporting the view that variation in this change induced by tax reforms is exogenous. Base-year income is, however, positively correlated with the change in predicted marginal net-of-tax rate. This suggests that the tax reforms in the 1980's gave individuals with higher base-year income a larger predicted marginal net-of-tax rate decrease. To sort out the effect of the change in predicted marginal net-of-tax rate at the base-year income from the direct effects of base-year income, therefore, relies on controlling for the independent effects of base-year income properly. The endogeneity of the change in marginal net-of tax rate at the base-year income, of course, does not necessarily indicate that variation arising from tax reforms is endogenous. More likely, it is a result of base-year income being endogenous.

#### 4.2 Broad income

It is commonly believed that the taxable income elasticity largely captures the effect of income shifting. We therefore report nonlinear elasticity estimates for broad income in Table 2, which is similarly organized as Table 1. The estimated broad income elasticity that accounts for nonlinear budget sets in the estimation is in the region of 0.20 and still quite insensitive to including a base-year income control function. In comparison, the estimated elasticity based on linearized budget sets is around 0.13 and is sensitive to including the control function. All estimates are statistically significant at the one percent level. This

reinforces our earlier result that accounting for nonlinear budget sets increases the estimated elasticity.

Table 2. Estimates of nonlinear elasticities of broad income

	(1)	(2)	(3)	(4)	(5)
Change log average marginal net-of-tax rate	0.126** (0.027)	0.145** (0.026)	0.173** (0.028)	0.202** (0.027)	0.199** (0.027)
Change log predicted marginal net-of-tax rate	-0.035** (0.012)	-0.017 (0.012)	-0.032** (0.012)	0.147** (0.012)	0.133** (0.012)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The change in the log of broad income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. \*  $p < 0.05$ ; \*\*  $p < 0.01$ .

In comparison, the estimated linear elasticity in Gruber and Saez (2002) were 0.12 and statistically insignificant. That corresponds to a statistically insignificant nonlinear elasticity of 0.06. Weber (2014) on the other hand obtained an estimated linear elasticity between 0.48 and 0.70 that translates to a nonlinear elasticity between 0.26 and 0.37. Therefore, our estimates lie within the range of the estimated elasticities in these two studies.

### 4.3 Difference length

The responsiveness of taxable income to tax rate changes may differ between the short-run and the long-run. On the one hand, it may take some time for individuals to react to tax changes rendering larger long-run effects. On the other hand, temporal income shifting between years (e.g., between December one year and January the following year) produces only short-run effects. In Table 3, we report nonlinear elasticity estimates that account for nonlinear budget sets in the estimation for taxable income and broad income using different difference lengths. We use observations with the same base-year, which keeps the number of observations constant.

Table 3. Nonlinear net-of-tax elasticity with different difference lengths

	(1) 1-year differences	(2) 2-year differences	(3) 3-year differences
Taxable income	1.250** (0.048)	0.788** (0.046)	0.748** (0.052)
Broad income	0.215** (0.022)	0.143** (0.022)	0.199** (0.027)

Notes: Each cell is a nonlinear consumption net-of-tax elasticity estimate from one regression. The change in the log of income is the outcome variable and the change in average marginal net-of-tax rate is the independent budget set variable. All specifications include controls for filing status, year dummies, and a spline in the log of base-year income. Each regression contains 51,392 observations. \*  $p < 0.05$ ; \*\*  $p < 0.01$ .

Table 3 shows that the estimated taxable income elasticity decreases with difference length, suggesting that there are some short-run effects. The estimated broad income elasticity

decreases first between the specifications with one- and two-year differences, but increases between the specifications with two- and three-year differences, suggesting that there are some effects with a longer response time.

## 5. Conclusion

We started with a standard taxable income model with a quasi-linear utility function. The estimated elasticity of taxable income with respect to the marginal net-of-tax rate corresponds to the elasticity of substitution parameter in the utility function when individuals face linear budget sets. We showed that, when allowing for nonlinear budget sets, the nonlinear consumption net-of-tax elasticity is a direct policy-relevant extension of the linear elasticity. We then showed that using linearized budget sets to estimate the linear elasticity when budget sets are nonlinear produces a downward bias unless a valid instrumental variable is used. We argued, however, that the structural (instrumental-variable) estimate of the linear elasticity cannot be used to consistently simulate policy effects of tax changes, while the reduced-form estimate of taxable income on the instrumental variable yields a policy-relevant nonlinear elasticity.

We extended the benchmark model to account for nonlinear budget sets and derived the taxable income function that depends on a weighted average of the marginal net-of-tax rates at every income level with the probability density function of unconditional income as weights. The estimated average marginal net-of-tax rate elasticity could also be interpreted as the nonlinear consumption net-of-tax elasticity. Since our specification accounts for the entire budget set, it circumvents the issues related to the observed marginal net-of-tax rate being endogenous to taxable income, such as mean reversion and widening income distribution, plaguing specifications based on linearized budget sets. We also highlighted a new source of endogeneity in such specifications arising from the omission of remaining marginal net-of-tax rates at other income levels than the observed income level.

In our empirical application, we estimated the nonlinear elasticity with respect to the average marginal net-of-tax rate using U.S. data from the NBER tax panel for 1979-1990 and differencing methods. We found an estimated nonlinear elasticity of 0.75 for taxable income and 0.20 for broad income. In comparison, the standard instrumental variable linearization method yielded a reduced-form estimate of the nonlinear elasticity with respect to the marginal net-of-tax rate at the base-year income of 0.27 for taxable income and 0.13 for broad income. Accounting for nonlinear budget sets therefore increased the estimated elasticity. Unlike the standard method, our estimated elasticity with respect to the average marginal net-of-tax rate was insensitive to controlling for base-year income, indicating that these estimates were unaffected by bias due to mean reversion or widening income distribution.

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## Appendix

Table A1. Sample statistics

Variables	Mean	Std. dev.	Min	Max
Change log taxable income	0.092	0.692	-4.022	4.275
Change log broad income	0.040	0.329	-2.265	2.885
Change log average marginal net-of tax rate	0.020	0.055	-0.142	0.186
Change log predicted marginal net-of tax rate	0.017	0.124	-3.005	4.902
Log base-year taxable income	2.905	0.937	0.000	6.520
Log base-year broad income	3.571	0.606	1.752	6.596
Log base-year average marginal net-of-tax rate	-0.431	0.061	-0.859	-0.175
Log base-year predicted marginal net-of-tax rate	-0.312	0.152	-3.297	0.877
Base-year taxable income	25.367	24.108	0.000	677.657
Base-year broad income	41.657	29.345	4.768	730.991
Base-year average marginal net-of-tax rate	0.655	0.039	0.424	0.847
Base-year predicted marginal net-of-tax rate	0.739	0.093	0.037	2.404

Notes: Taxable income and broad income are in USD at the 1990 price level.

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