UPPSALA UNIVERSITET

Uppsala Center for Fiscal Studies

Department of Economics

Working Paper 2012:6

Gender-Based and Couple-Based Taxation

Spencer Bastani

Uppsala Center for Fiscal Studies
Department of Economics
Uppsala University
P.O. Box 513
SE-751 20 Uppsala
Sweden
Fax: +46 18 471 14 78

Working paper 2012:6
March 2012

Gender-Based and Couple-Based Taxation

Spencer Bastani

Gender-Based and Couple-Based Taxation

Spencer Bastani*

March 28, 2012

Abstract

In a recent paper Alesina et al. (2011) construct a model in which different labor supply elasticities for men and women emerge endogenously from intra-household bargaining. In this paper I explore the optimal tax implications of their model in an economy with both singles and couples and inequality across as well as within households. In the model, the welfare of married women can be improved by lowering taxes for single women. Moreover, if single men earn more than single women, the welfare of married women can alternatively be improved by a gender-neutral tax scheme which taxes singles at a higher rate. Because the government is concerned not only with equalizing utilities within families, but also with the redistribution between high income and low income households, gender-based adjustments in the income tax must be weighed against the welfare consequences of changing the progressivity of the tax system. I find that larger lump-sum transfers to women is always optimal. Interestingly, marginal tax rates, on the other hand, should be lower for women only if the exogenous bargaining power of men is moderate. The welfare gains of gender based taxation are sizable and the welfare gains of having tax instruments which depend on household composition are even larger.

Keywords: optimal taxation; tagging; family economics; intra-household bargaining

JEL Classifications: D13, H21, J16, J20

*Department of Economics, Uppsala University and Uppsala Center for Fiscal Studies, Box 513, SE-751 20 Uppsala, Sweden. E-mail address: spencer.bastani@nek.uu.se. I am grateful to Dan Anderberg, Ted Bergstrom, Sören Blomquist, John Conley, Luca Micheletto, Eva Mörk, Katarina Nordblom, Håkan Selin and Laurent Simula as well as seminar participants at Uppsala University and the 2011 Symposium of Uppsala Center for Fiscal Studies for helpful comments and suggestions.

1 Introduction

In a recent paper Alesina et al. (2011) analyze gender-based taxes as a potential tax policy. Their argument is based on the labor supply behavior of men and women being different. Two sources of these differences are explored. Either women have a comparative advantage in household production, or, for cultural reasons, the intra-family bargaining process is favoring the husband. In both cases, women pursue more home duties and work less in the market. Because of this, a higher labor supply elasticity for women emerges endogenously from intra-household bargaining. The authors then show that labor market distortions can be reduced if women are subject to a lower marginal tax rate according to Ramsey's rule.

The goal of the present paper is to expand upon the analysis of Alesina et al. (2011) in several directions and explore gender-based taxation in greater detail. In the concluding remarks of Alesina et al. (2011) the authors suggest, "The tax policy part of our model would benefit from an analysis closer to that of Mirrlees as the policy of differential taxation by gender redistributes not only within households but also across households." For natural reasons, as they consider a representative agent of each gender, lump-sum transfers play a trivial role because they can be used to raise revenue and redistribute wealth across genders at no efficiency cost. The present paper considers a distribution of taxpayers of each gender, creating a countervailing motive to redistribute across households of different productivity types.

Alesina et al. (2011) analyze gender-based taxes which apply irrespective of marital status. A central part of the model they consider is that couples Nash-bargain over the distribution of resources in the household. The bargaining power of spouses is affected by the outside options, determined by the utility of single individuals. However, singles are not considered as separate economic agents in their model. The authors conjecture that if tax rates differed not only across genders but also within genders, the government could directly offset the bargaining power of men by taxing single men at a higher rate. In this paper I explore this mechanism in a model with both singles and couples and show that the benefit of taxing single men at a higher rate in order to increase the bargaining power of married women must be weighed against the inequality induced between single men and single women.

While Nash-bargaining with divorce threat points is a well-analyzed model of intra-household decision-making, dating back to the influential contributions of Manser and Brown (1980) and McElroy and Horney (1981), Lundberg and Pollak (1993) propose an alternative Nash-bargaining model. They suggest that for many couples the relevant threat point for the bargaining solution is not divorce, but an "inefficient noncooperative equilibrium within marriage". I pursue this idea and explore the robustness of gender-based taxation when threat points are determined by a game where each spouse contributes voluntarily to household public goods and maximize their own utility, given the actions of their spouse.

When spouses bargain under the threat of divorce, who receives or controls income within marriage has no consequence for the Nash-bargaining solution. In contrast, when threat points are internal to marriage, the intra-household allocation varies with the source of income within the household. Thus, if threat points are internal to the marriage, family bargaining suggests tax rates should differ by gender for married individuals. I find that the introduction of this alternative threat point specification does not affect the conclusions regarding the structure of gender-based taxation, but has important consequences for a gender-based tax system differentiating between singles and couples.

I study the problem of a social planner aiming to address the inequality in the economy as measured by a concave social welfare function defined over individual utilities. The planner has access to linear income taxation of the form analyzed by Sheshinski (1972). Under a linear income tax an individual receives a lump-sum transfer and pays a proportional tax rate on all positive income. The linear tax framework is closely related to that of Mirrlees and enables the exploration of gender-based taxation with bargaining couples and redistribution across households without running into the difficulties of multi-dimensional screening problems.

I consider four different tax systems: (i) a benchmark optimum, which applies a uniform linear income tax to all individuals, (ii) a gender-based tax which does not distinguish between singles and couples, (iii) a tax which is gender-neutral but taxes singles and couples differently (a "couple-based" tax) and finally, (iv) a tax system which uses information on both gender and couple-status. This tax system, referred to as the Full Optimum, applies separate tax schedules to the four groups of individuals in my model; single men, single women, men in couples and women in couples. With these tax instruments at its disposal, the government is essentially facing a tagging problem, as analyzed theoretically by Boadway and Pestieau (2006) and more recently by Cremer et al. (2010). In a tagging optimum the average social marginal utility of income is equalized across groups by means of lump-sum transfers while tax rates are differentiated so that an equitable distribution of consumption within each group emerges. However, due to intra-household bargaining, the standard motive of redistributing from high income to low income households must be weighed against the effects on the intra-household distribution of resources. Because the utility of singles enter the equilibrium utility of married men and women, the formula for optimal income taxation contain additional terms reflecting an external effect on the intra-household distribution within couples imposed by the tax treatment of singles.

In the present paper, because I consider a setting with heterogeneity in market ability, I am able to explore the conditions under which it is optimal to adopt gender-based lump-transfers and to which extent tax rates should be differentiated according to gender.[1] An interesting finding is that when the parameter governing the exogenous bargaining power of men is high, it is no longer the case that the tax rate on female earnings should be lower. The intuition for this is that an increase in earned income as a result of an increase in the labor supply of the woman by a decrease in her tax rate will be unevenly distributed. Instead, by increasing the lump-sum transfer to women, the income of the family can be raised and the bargaining power of the woman can be strengthened without her labor supply being increased.

To characterize optimal taxes and assess the welfare effects of differential taxation I undertake a mainly quantitative investigation of optimal tax policy. Because the linear income tax contains a lump-sum component which is likely to induce an income effect on labor supply, to obtain measures of welfare gains relevant for real economies, I consider a model with nonlinear utility of consumption in my numerical simulations. The model has labor supply implications which are attractive from an empirical perspective and well-suited for a quantitative investigation. First, in accordance with Alesina et al. (2011), the presence of household production generates a higher elasticity of labor supply for women. In my model, these gender-differences in labor supply are generated solely by differences in market wages between men and women. Alesina et al. (2011) show that gender-differences in wages can be generated by men and women pursuing different careers. In this paper I abstract from career decisions by taking these wage differences as exogenous. This reduced form approach simplifies the analysis and does not require the specification of a career cost function, an object which is little known about empirically. Second, because I allow for wage heterogeneity and adopt a specification where each individual is equally productive in household work, the elasticity is also higher at the bottom of the earnings distribution, consistent with empirical evidence.

I calibrate the model to agree with empirical facts using time-use data from the American Household Survey and wage distributions for each gender are parameterized using Swedish register data. I verify that the wage distribution for women lies to the left of the wage distribution for men at each wage percentile. In addition the female wage distribution is more compressed. As Sweden sometimes is regarded as an 'extreme' in terms of wage equality, the scope for redistribution and correspondingly the welfare gains obtained in this paper, are likely to be higher for other countries.

[1] In contrast, due to the setup of their model, Alesina et al. (2011) only analyze the role of differentiated marginal tax rates.

Finally, although the main contribution of the paper is to explore gender-based taxation in greater detail, the model sheds light on how singles should be taxed relative to couples, an important question which has received surprisingly little attention in the optimal tax literature. Traditional analysis has either focused exclusively on couples or simply assumed that single and couple households act and derive utility according to the same economic model.

My results can be summarized as follows. I find that gender-based taxation is associated with welfare gains ranging from 0.28% to 0.81% of aggregate income depending on the value of the parameter governing the bargaining power of men in the household. Another significant determinant of the welfare gains of gender-based taxation is the gender wage-gap. I find that if the gender wage-gap is 5% larger than in Sweden, the welfare gains of gender-based taxation exceed 2%. The results regarding the structure of gender-based taxation are robust to perturbations in the modeling framework. More specifically, I obtain qualitatively similar results when the Nash-bargaining threat points are determined by a non-cooperative game within marriage as compared when these threat-points are determined by the utility of single individuals. The welfare gains of allowing the tax authority to distinguish between singles and couples are large. These welfare gains range between 0.44% and 2.73%. The source of the benefit from couple-based taxation is the redistribution from couple to single households which depends on the importance of economies of scale in couple households.[2] The result stands in sharp contrast to the current tax practice in the United States.[3] The welfare gain of taxes which depend on both gender and marital status, in the 'Full Optimum', is about the sum of the respective welfare gains taken together. An interesting finding is that the 'conventional wisdom' that the male tax rate should be higher than the female tax rate is reversed when the bargaining power of men is high.

The paper is organized as follows. In section 2 I derive a modified version of the Alesina et al. (2011) household model without income effects allowing for wage heterogeneity and the explicit introduction of singles. In this case readily interpretable formulas for the optimal linear income tax can be derived (presented in section 3). In section 4 I extend the baseline model to the case of income effects on labor supply (section 4.1) and introduce the model with non-cooperative threat points (section 4.2). I then discuss data and calibration strategies (sections 4.3 and 4.4) and turn to the main contribution of the paper which is the quantitative analysis of optimal tax policy. The results are discussed in section 5 and section 6 offers concluding remarks.

[2] Although the analysis abstracts from "family needs" which arise due to the presence of children in the households.

[3] Alm et al. (2002) find clear evidence of a 'singles tax' in the 2002 US Tax Structure.

1.1 Related literature

There is a growing literature on tagging and optimal income taxation. Recent contributions have analyzed taxes which depend on an individuals' age, height or the number of children in the household.[4] Furthermore, the welfare effects from tagging have been found to be quantitatively important (Bastani et al. (2010b), Bastani et al. (2010a), Weinzierl (2011)).

The idea of a gender-based tax dates back to at least Rosen (1977) and was formally developed by Boskin and Sheshinski (1983) who established the 'conventional wisdom' that the deadweight loss of taxation could be made smaller if women were subject to a lower marginal tax rate. More recently Apps and Rees (2011) have revisited the analysis of Boskin and Sheshinski. Apps and Rees point out that a higher elasticity of female labor supply is not sufficient to conclude that men should be taxed at a higher rate. They emphasize the importance of considering the effectiveness of the male and female tax rates in redistributing from high income to low income families (i.e. whether male or female earned income is more highly correlated with the welfare of the family). Another related paper is Viard (2001) which is an interesting contribution studying tagging in the context of linear income taxation.

The practice of using numerical simulations in analyzing linear taxation dates back to Stern (1976). Boskin and Sheshinski (1983) also used a numerical simulation model similar to the one used here to analyze gender-based taxation. However these authors considered a unitary model of the household where intra-household inequality is absent. Further, the welfare effects of differential taxation were not assessed. Cremer et al. (2010) is a recent contribution which explores tagging according to gender in the quantitative part of their paper. Although they allow for fully nonlinear taxation, they do not consider a family model. In their setting gender-based taxation is welfare improving because with separate nonlinear income tax schedules at its disposal, the government can tailor income tax rates to the skill distribution of each gender.

2 The model

My model of the household builds closely on Alesina et al. (2011). The main differences as compared to their work are that I allow for both singles and couples and consider populations of agents with different wage rates. I also abstract from career decisions and consider income effects on labor supply in section 4.

The economy is populated by men and women living together as a couple or living alone as

[4] An early example is Immonen et al. (1998). Other recent examples include Boadway and Pestieau (2006), Blomquist and Micheletto (2008), Mankiw and Weinzierl (2010), Blundell and Shephard (2011) and Weinzierl (2011).

singles. Each agent is characterized by his/her productivity type $i \in \{1,..,N\}$, gender $j = m, f$ and couple status $k = s, c$ (where s indicates that an individual is living in a single household and c denotes an individual living in a couple household). Thus agents are indexed by the set $\mathcal{S} = \{1,..,N\} \times \{m, f\} \times \{s, c\}$. A variable x associated with an individual $(i, j, k) \in \mathcal{S}$ is denoted x_{ij}^k. Whenever it is obvious, the superscript k is suppressed.[5]

Denote by π_i the fraction of productivity type i in the population satisfying $\sum_{i=1}^N \pi_i = 1$. There is an equal mass 1/2 of men and women of each productivity type. A fraction η of all agents are singles and a fraction $1 - \eta$ are living as couples. The total population size is normalized to 1. The productivity type of an agent determines his/her market wage rate drawn from a discrete gender-specific wage distribution $\{(\pi_i, w_{ij})\}_{i=1}^N$, $j = m, f$ where $\{w_{ij}\}_{i=1}^N$ is a non-decreasing sequence of wage rates. The couple-status of an individual does not affect his/her wage rate.[6]

To analyze couples in the presence of heterogenous earnings abilities, the matching process needs to be specified. I assume exogenous, perfect assortative matching. In the model this means that a male and a female agent with the same productivity type forms a couple. The couple is simply referred to as "couple i" for all $i \in \{1, \ldots, N\}$ and its members have wage rates w_{im} and w_{if}. Due to the assumptions, any couple $j > i$ has unambiguously a higher earnings capacity than couple i. [7]

In the economy there are two types of work. Each agent supplies labor along the dimensions market work ℓ and household work h.[8] One unit of market labor effort ℓ earns the unitary wage rate w. Following Alesina et al. (2011), work effort in the household sector is transformed into household goods through a decreasing-returns-to-scale production technology. More specifically, letting H represent the household good, I assume

$$H_{ij}^s = F_{ij}(h_{ij}^s) = \frac{\omega^s}{\alpha^s}(h_{ij}^s)^{\alpha^s} \tag{1}$$

[5]In the paper I use the words 'couple' or 'married' interchangeably. When referring to the members of a couple I sometimes use the words 'husband' and 'wife' or 'spouse'.

[6]Keep in mind that $w_{im} \neq w_{if}$ reflects that i is an index describing an individuals' *relative* position in the productivity distribution pertaining to his/her gender.

[7]The matching process has potentially interesting implications for income taxation. Perfect assortative matching seems however a reasonable benchmark. In the quantitative section I mimic a situation with positive matching by scaling up the wages of all men by a given constant.

[8]The primary focus in this paper is on average behavior, hence the exclusion of the extensive margin of labor supply is inessential.

and

$$H_i = F_i(h_{im}, h_{if}) = \frac{\omega^c}{\alpha^c}\left[(h_{im})^{\alpha^c} + (h_{if})^{\alpha^c}\right] \tag{2}$$

where $F_{ij}(h^s_{ij})$ is the production function for singles and $F_i(h_{im}, h_{if})$ is the corresponding (joint) production technology for couples. The parameter sets $\{\omega^s, \alpha^s\}$ and $\{\omega^c, \alpha^c\}$ describe the shape of the production technologies and satisfy $\omega^k > 0$ and $0 < \alpha^k < 1$, $k = s, c$.[9] All individuals are equally productive in household work but the marginal product of one unit of household work is different depending on if it is supplied within a single or couple household. The assumption underlying the couple's production function is that spouses' household work are imperfect substitutes. [10]

2.1 Individual optimum

I consider a two-good economy where individual preferences are represented by a utility function assumed to be quasi-linear in consumption and household goods. This means that goods can either be bought on the market, or produced at home.[11] The assumption that market goods and household goods are perfect substitutes is in line with Gronau (1977) and the following empirical literature. This assumption seems reasonable when considering goods such as cleaning, food preparation and repairs.

Singles choose ℓ^s_{ij} and h^s_{ij} to maximize

$$U^s_{ij} = c^s_{ij} + H^s_{ij} - \frac{1}{1+\phi}(\ell^s_{ij} + h^s_{ij})^{1+\phi} \tag{3}$$

subject to their budget constraint $c^s_{ij} = (1-\tau^s_j)w_{ij}\ell^s_{ij} + G^s_j$ and technology constraint $H^s_{ij} = \frac{\omega^s_i}{\alpha^s}(h^s_{ij})^{\alpha^s}$. The first order conditions for singles imply

$$h^s_{ij}: \quad \omega^s(h^s_{ij})^{\alpha-1} = \left(\ell^s_{ij} + h^s_{ij}\right)^{\phi}$$
$$\ell^s_{ij}: \quad (1-\tau^s_j)w_{ij} = \left(\ell^s_{ij} + h^s_{ij}\right)^{\phi}$$

[9] As in Apps and Rees (1997) only time enters as an input to the household production. Introducing also consumption goods in the production process would only complicate the analysis with no substance for the results.

[10] The importance of household production in the analysis of two-person couples has been recognized ever since the seminal contribution of Becker (1965). Time not devoted to market work cannot be viewed as pure leisure as that would give an incomplete picture of the process generating utility. This is especially relevant when analyzing couple households where household work may be distributed unevenly among the spouses.

[11] In order to obtain closed-form expressions for the effects of taxes on the utility of members of couple households, it is necessary to abstract from income effects on labor supply. In the quantitative part of the paper the quasi-linearity assumption is relaxed.

hence household work is given by the function

$$h_{ij}^{s}=\left(\frac{\omega^{s}}{(1-\tau_{j}^{s})w_{ij}}\right)^{\frac{1}{1-\alpha^{s}}}$$

The specified technology implies that all agents will engage in at least some household work. Singles increase their work effort until the marginal product of household work equals their after-tax wage rate. Hence the price of the domestic good is exogenously fixed by the market. Because the individual-specific household productivity parameter ω^s is constant across types and market wage rates increase in i, higher skill individuals work more in the market and less at home (consistent with empirical evidence).

Singles and couples have identical preferences. The preferences of members of couple households are represented by the utility function

$$U_{ij}=c_{ij}+H_{i}-\frac{1}{1+\phi}(\ell_{ij}+h_{ij})^{1+\phi}\quad j=m,f. \tag{4}$$

The budget constraint for the family is

$$\sum_{j=m,f}c_{ij}=z\sum_{j=m,f}\left((1-\tau_{j})\ell_{ij}w_{ij}+G_{j}\right). \tag{5}$$

The distinguishing feature of the couple is that there is a 'surplus' or 'economies of scale' because many goods produced at home or bought in the market benefit both spouses. Goods produced at home are assumed to be pure public goods while goods bought in the market are assumed to represent a mixture of private and public goods. The public good aspect of market goods is captured in a simple way in (5) by assuming that each dollar of income in the couple generates a value of goods and services which is equal to z where $z\in(1,2)$.[12]

The feasible set of intra-household allocations (U_{im},U_{if}) is determined by the technology (2), the family budget constraint (5) and the utility function (4). To pin down the intra-household equilibrium, the family decision-process needs to be specified. From the perspective of optimal tax analysis, the simplest assumption to make is that the allocation is determined by a 'benevolent household dictator' with the same inequality aversion as the government. In this case intra-household inequality has no implications for optimal taxation. However, this 'unitary' model of household decision-making is nowadays regarded to have little empirical support.[13] In this paper I adopt the Nash-bargaining model and assume that spouses bargain over the intra-household allocation in the spirit of Manser and Brown (1980) and McElroy and Horney

[12]If $z=1$ the market good would be 'purely private' and if $z=2$ the good would be 'purely public'. This is a reduced form approach. Manser and Brown (1980) explicitly introduce both private and public market goods. In their model, "love or companionship" amplifies the utility gains from private consumption in a similar fashion.

[13]See Chiappori (2009) for an overview. Lise and Seitz (2011) find substantial inequality within UK households.

(1981). In this model the bargaining power is determined by reference to the utility each spouse would enjoy as single. Following Alesina et al. (2011) I introduce the parameter γ reflecting the culturally inherited bargaining power of men. This parameter introduces asymmetry in the Nash-bargaining solution and captures factors which, due to the external economic environment, history or culture, might disfavor specific members of the household. The asymmetry in the Nash-Bargaining solution has strong implications for the distributional consequences of transfers to the household. Somewhat surprisingly, the asymmetric solution has received surprisingly little attention in the literature on family decision-making.

The household allocation is determined by the maximization of the product of the gains from marriage, [14]

$$\max_{\{c_{ij},\ell_{ij},h_{ij}\}_{j=m,f}} \Omega_i = (U_{im} - U_{im}^s)^\gamma (U_{if} - U_{if}^s)^{1-\gamma} \tag{6}$$

subject to the household budget constraint (5), the household production constraint in (2) and the participation constraints $U_{im} \geq U_{im}^s$ and $U_{if} \geq U_{if}^s$.[15] Let $y_i = \sum_{j=m,f} \left[(1-\tau_j)\ell_{ij}w_{ij} + G_j\right]$ denote total family income for a couple household. The quantity

$$Q_i \equiv U_{im} + U_{if} = zy_i + 2H_i - \frac{1}{1+\phi} \sum_{j=m,f} (\ell_{ij} + h_{ij})^{1+\phi}$$

is household profit, defined as the total value of goods and services consumed by the household, minus total production costs. In appendix B it is shown that the equilibrium utility of each spouse under Nash-bargaining is given by:

$$U_{im} = U_{im}^s + \gamma(Q_i - U_{im}^s - U_{if}^s) \tag{7}$$

and

$$U_{if} = U_{if}^s + (1-\gamma)(Q_i - U_{im}^s - U_{if}^s) \tag{8}$$

Thus players can be seen as bargaining over the 'marital pie' Q_i. First, both spouses agree to give each other the threat point utilities U_{im}^s and U_{if}^s (which they would achieve if they would not reach agreement) then the man receives a fraction γ, and the woman, a fraction $1-\gamma$, of

[14] Notably, Nash-Bargaining assumes efficiency. When $\gamma \neq 1/2$ this is referred to as the generalized Nash-bargaining problem which does not satisfy the axiom of symmetry. Concave utility over contracts guarantees that the set of feasible utility possibilities (U_m, U_f) is convex and the Nash-Product is quasi-concave. An alternative solution concept is the Kalai-Smorodinski (KS) bargaining solution. While Nash-Bargaining requires that the utility possibilities set is convex, KS requires only that it is comprehensive. However the KS solution does not satisfy 'independence of irrelevant alternatives' which is a desirable property of the Nash-Bargaining solution.

[15] I assume the surplus within marriage is sufficiently large so that the participation constraints are satisfied.

the remaining surplus $(Q_i - U_{im}^s - U_{if}^s)$. Since the surplus is positive, an increase in γ always makes the woman worse off and the man better off.

2.2 Tax instruments

In this paper I allow for income taxes of the form $T(y) = -G + \tau y$ and refer to such a tax schedule as (τ, G). The income tax paid is allowed to depend on non-income characteristics. More specifically, in addition to the simple uniform linear tax schedule (τ, G) I consider the following tax systems: (i) gender based tax schedules $\{(\tau_j, G_j)\}_{j=m,f}$, (ii) couple-based tax schedules $\{(\tau^k, G^k)\}^{k=s,c}$, and (iv) the 'full optimum' consisting of the set $\{(\tau_j^k, G_j^k)\}_{j=m,f}^{k=s,c}$.[16] The question analyzed here is how these tax instruments can be used to address the inequality in the economy.

Before formally stating the problem of the government it is useful to analyze the distributional consequences of various changes to the tax systems. These distributional consequences can be divided into two categories. First, the tax systems will affect the intra-household allocation within couple households because differentiated taxes change the bargaining power of spouses. Second, with a linear income tax, the average tax paid is different for low income and high income households, hence the allocation of consumption across households is affected. This means that, in general, the government faces a trade-off between redistributing income within and across households.

2.3 The effects of taxes on household welfare

It is straightforward, by the envelope theorem, to derive the effect on the utility of singles as a result of a change in the tax rate τ or the lump-transfer G. Let V_{im}^s and V_{if}^s denote the indirect utility functions corresponding to (3). Insertion of the budget constraint into (3) and taking derivatives yields immediately $\frac{\partial V_{ij}^s}{\partial \tau} = -w_{ij}\ell_{ij}^s \equiv -y_{ij}^s$ and $\frac{\partial V_{ij}^s}{\partial G} = 1$ for $j = m, f$.

For couples, equations (7) and (8) are differentiated with respect to the various taxes τ and G to obtain the marginal effect of taxes on individual utility. The complete list of partial

[16] The problem of designing a progressive linear income tax in the presence of inequality in earnings capacities is closely related to the problem of income taxation pioneered by Mirrlees (1971). In the general problem of optimum income taxation with a set of potential tags Θ, the tax-transfer system is captured by a general nonlinear function of labor income $T(Y;\Theta)$ where Θ is a set of demographic characteristics (such as family-status, gender, number of children etc.) and Y is labor earnings. In this paper I restrict attention to an affine approximation of this function e.g. for $\theta \in \Theta$, $T(Y) = -G_\theta + \tau_\theta Y$ where G_θ is a lump sum transfer. When $G_\theta > 0$ this is a progressive income tax. The introduction of non-income characteristics is an important source of non-linearity in actual tax-benefit schemes. Ooghe and Peichl (2010) report that on average, around 50% of the variation in taxes among EU countries is explained by non-income characteristics.

derivatives admitted by tax systems (i)-(iii) are given in table 7 of appendix B. Here I focus on the main results.

To understand the effects on the within-household distribution of resources it is helpful to begin analyzing changes in lump-sum transfers because these affect the bargaining power of spouses in households with different incomes in the same way. Equations (7) and (8) are key to understanding how the tax instruments affect the intra-household allocation. These equations show that a tax reform which results in a \$1 increase in the income of the family (through an increase in Q_i) will benefit married men by $\$\gamma$ and benefit married women by $\$(1-\gamma)$. If $\gamma > 1/2$, the husband gains more from this reform than the wife.[17] Thus, when the exogenous bargaining power of the husband is strong, increasing transfers to the family will be an inefficient way to increase the utility of the wife.

Because the utility of singles enter the equilibrium utility of couples in equations (7) and (8), taxing single men and women differently will have asymmetric effects on the utility of spouses. A gender-based tax reform which increases the utility of single women by \$1 with an equal reduction in the utility of the single men, will increase the utility of the wife in the couple household by enabling her to claim an additional full \$1 from her husband in the bargaining process. This reform will reduce any pre-existing inequality between married men and women, at the expense of exacerbating inequality between single men and women.

However, even a completely gender-neutral reform can have an asymmetric impact on the utility of spouses and potentially increase the welfare of married women in this model. Consider a reform which raises both U^s_{if} and U^s_{im} by $\$\frac{1}{2}$ and thus shrinks the marital surplus $(Q_i - U^s_{im} - U^s_{if})$ by \$1. One might think that this reform does not affect the distribution within households because both threat points are affected by the same amount. This is correct if $\gamma = 1/2$. However, if $\gamma \neq 1/2$ then the marital surplus is distributed unevenly and any increase or reduction in this surplus will have distributional consequences. Suppose $\gamma > 1/2$, then a gender-neutral tax reform which redistributes \$1 from married to singles (each single gaining $\$\frac{1}{2}$) will have a positive impact on the welfare of married women. This gain will be equal to $\$(-1+2\gamma)$.

The discussion above focused on the externality of the tax treatment of singles on the utility of couples as a result of various changes in lump-sum transfers. These are income-independent ways of changing the bargaining power of spouses. Another way of affecting the bargaining power in a way which depends on the income of agents is to employ differentiated marginal tax rates. This is the subject of section 3.1 where I discuss optimal gender-based tax rates and section 3.2 where I discuss gender-neutral taxation which taxes the income of singles and couples

[17] Analogously, a \$1 reduction in the income of the family will only reduce the utility of the wife by $\$(1-\gamma)$.

at different rates.

Nash-Bargaining with divorce-threat points is by far the most commonly analyzed bargaining model of family decision-making. An alternative class of models proposed by Lundberg and Pollak (1993) and surveyed by Lundberg and Pollak (1996) are based on the idea that the relevant threat point is not divorce but instead a 'non-cooperative game' within marriage. In section 4.2 I apply this concept in the simplest possible way by considering an alternative specification of threat points which are 'internal' to marriage and determined by the distribution of utilities which would prevail if spouses behave non-cooperatively within the marriage or couple. The important consequence of adopting internal threat points is that, in contrast to the situation when the threat points are determined outside the couple, the identity of the person in the household who receives transfers or controls income within the marriage determines the intra-household distribution. Effectively, this exercise 'shuts down' the externality of the tax treatment of singles on the distribution of utility within couples.

3 Optimal taxation

The purpose of this section is to derive formulas for optimal linear income taxation. I first derive and present the formula for the optimal linear income tax in a form which conforms to the classic analysis of Sheshinski (1972). Then I proceed to analyze gender-based taxation, couple-based taxation and the full optimum. To obtain formulas expressed in terms of averages across the whole population, I introduce π_{ij}^k as the fraction of agent $(i, j, k) \in \mathcal{S}$ in the population where $k = s, c$ indicates single and couple households respectively. As before, η and $(1 - \eta)$ are the fractions of singles and couples in the population.[18] In the unrestricted 'full optimum', the government solves

$$\max_{\{\tau_j^k, G_j^k\}_{j=m,f}^{k=s,c}} \sum_{ijk} \pi_{ij}^k W(V_{ij}^k) \tag{9}$$

subject to the government budget constraint

$$\sum_i \pi_i \sum_{j=m,f} \frac{1}{2} \left[\eta \tau_j^s w_{ij} \ell_{ij}^s + (1-\eta)\tau_j w_{ij} \ell_{ij}\right] \geq \sum_{j=m,f} \frac{1}{2} \left[\eta G_j^s + (1-\eta) G_j\right] + R \tag{10}$$

where R is an exogenous revenue requirement and W is a concave function representing the government's taste for redistribution.

Suppose the government is constrained in its tax policy and chooses a single tax rate τ and a transfer G. The first order conditions for τ and G following maximization of (9) subject to

[18] Recall that the total population size is normalized to 1 and there is an equal mass of men and women in the population.

(10) are

$$\tau : \sum_{ijk} \pi_{ij}^k \left(W'(V_{ij}^k) \frac{\partial V_{ij}^k}{\partial \tau} \right) = \sum_{ijk} -\pi_{ij}^k \lambda \left(y_{ij}^k + \tau \frac{\partial y_{ij}^k}{\partial \tau} \right) \tag{11}$$

$$G : \sum_{ijk} \pi_{ij}^k \left(W'(V_{ij}^k) \frac{\partial V_{ij}^k}{\partial G} \right) = \lambda \tag{12}$$

As is customary in the optimal tax literature, I define $\vartheta_{ijk} = \frac{W'(V_{ij}^k)}{\lambda} \frac{\partial V_{ij}^k}{\partial G}$ as the 'social marginal utility of income' for an (i, j, k)-agent where λ is the multiplier on the government budget constraint (10). The quantity ϑ_{ijk} measures the value to the government of a \$1 increase in the disposable income of agent (i, j, k).[19] Letting E denote the expectations operator, equation (12) can be written $E[\vartheta] = 1$.[20] In other words, optimality requires the average value of a \$1 increase in income to all agents to be equal to its cost (which is \$1 because the population size is normalized to 1). Using this result, expression (11) can be solved for τ and written as:

$$\tau = \frac{\mathbf{cov}(\vartheta, y)}{\overline{y}'}, \tag{13}$$

where $\overline{y}' = \sum_{ijk} \pi_{ij}^k \frac{\partial y_{ij}^k}{\partial \tau}$ is the average compensated earned income response. This is the standard formula for a linear income tax without income effects as expressed by Atkinson and Stiglitz (1980) and others. When the government is concerned with redistributing income from high ability to low ability individuals, the covariance between y and ϑ is negative. The denominator is the average (Hicksian) earned income response which is negative (following utility maximization in the standard model). Hence the marginal tax rate is positive and the tax revenue is used to finance the positive demogrant G.

3.1 Gender-based taxation

Suppose now the government has access to gender-based taxation, assigning marginal tax rates τ_m and τ_f in order to finance the transfers G_m and G_f. I first derive the optimality criterium for gender-based transfers. For this purpose, define $\vartheta_i^c = z\left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma)\frac{W'(V_{if})}{\lambda}\right)$ as the social marginal utility of couple i. This represents the benefit to society of increasing the disposable income of the couple by \$1. Denote also by $\vartheta_{ij}^s = \frac{W'(V_{ij}^s)}{\lambda}$ the social marginal utility of single (i, j). The first order conditions following maximization of (9) w.r.t. G_m and G_f are (details

[19]Remember that I am considering the case where utility is quasi-linear in consumption so that there are no income effects on labor supply.

[20]The use of statistical operators in this fashion has become a standard way of expressing averages in the linear income tax literature.

are provided in appendix D)

$$G_m : \sum_i \pi_i \left(\eta \vartheta_{im}^s + (1-\eta)\vartheta_i^c + (1-\eta)(1-\gamma)\frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = 1 \tag{14}$$

$$G_f : \sum_i \pi_i \left(\eta \vartheta_{if}^s + (1-\eta)\vartheta_i^c - (1-\eta)\gamma\frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = 1 \tag{15}$$

By subtracting the second from the first equation above and rearranging terms the following proposition can be established:

Proposition 1 (Gender-Based Transfers) *Optimal gender-based transfers are characterized by the condition*

$$\sum_i \pi_i \frac{\eta W'(V_{if}^s) + (1-\eta)W'(V_{if})}{\lambda} = \sum_i \pi_i \frac{\eta W'(V_{im}^s) + (1-\eta)W'(V_{im})}{\lambda} \tag{16}$$

The proposition above states that the average social marginal utility of income should be the same for men and women. The variation in taxes underlying the above expression is an increase in the transfer to one gender of \$1 combined with a reduction in the transfer to the other gender by \$1. This causes a differentiation in the lump-sum transfers received by single men and women with an associated change in the outside options for married men and women. The income of couple households and the marital surplus is not affected because the sum $G_m + G_f$ does not change.

For instance, if G_f is increased and G_m is decreased, the utility of single women will increase and the utility of single men will decrease. However, there is also an "externality" in the sense that there will be an improvement of the within-household distribution in couple households where the husband is better off than the wife. For this reason, it might be the case that transfers should be differentiated beyond the point where the average social marginal utility of income for *single* men and women are equalized. Up to a certain point, a more generous lump-sum transfer to women always reduces (average) inequality between single men and women and causes an improvement of the within-household distribution. Beyond this point, a more equitable within-household distribution can only be achieved by exacerbating the inequality between single men and women. This will be desirable when there is relatively more pre-existing inequality between men and women in couple households as compared to the inequality between single men and women.

In an economy with only singles ($\eta = 1$) proposition 1 can be seen as a special case of the result in the tagging literature which states that if the government is allowed to employ different lump-sum taxes for different groups of individuals, these lump-sum transfers should be designed so that the average social marginal utility of income for each group is the same.[21] Furthermore,

[21]See Viard (2001) for the case of linear taxation and Cremer et al. (2010) for the case of nonlinear taxation.

it has been shown that lump-sum transfers should be higher for groups which are relatively disadvantaged as compared to other groups.[22] The question of interest here is whether lump-sum transfers should be higher for men or women. For the general case $\eta \in [0,1]$ a sufficient condition for $G_f > G_m$ is $(1-\tau_m)w_{im} > (1-\tau_f)w_{if}, \forall i$.[23] This condition clearly holds if $\tau_m \leq \tau_f$ since I have assumed that men have higher wages than women at each percentile of the wage distribution. In section 5, the quantitative analysis demonstrates that $G_f > G_m$ is indeed a feature of the optimum. This result is robust across all model specifications considered in this paper and over a wide range of parameter values.

The next proposition highlights three considerations which determine gender-based marginal tax rates.

Proposition 2 (GBT, marginal tax rates) *Under a gender-based tax scheme the optimal marginal tax rates are given by*

$$\tau_m = \frac{1}{\overline{y}'_m} \sum_i \pi_i \Big[\eta \left(\vartheta^s_{im} - 1 \right) y^s_{im} + (1-\eta) \left(\vartheta^c_i - 1 \right) y^c_{im} + (1-\eta)(1-\gamma) \left(\frac{W'(V_{im}) - W'(V_{if}))}{\lambda} \right) y^s_{im} \Big]$$

and

$$\tau_f = \frac{1}{\overline{y}'_f} \sum_i \pi_i \Big[\eta \left(\vartheta^s_{if} - 1 \right) y^s_{if} + (1-\eta) \left(\vartheta^c_i - 1 \right) y^c_{if} - (1-\eta)\gamma \left(\frac{W'(V_{im}) - W'(V_{if}))}{\lambda} \right) y^s_{if} \Big]$$

Proof. See appendix D. ■

Apart from the third term in brackets, the optimal tax formulas above have the same structure as (13). As before, the denominator is the average (compensated) earned income response (here across ability types, singles, and couples). The result of numerous empirical studies suggest $\overline{y}'_m < \overline{y}'_f$ (primarily driven by the labor supply response of married women, see Blundell and MaCurdy (1999) for a review). Thus this efficiency consideration pulls the formulas in the direction of $\tau_f < \tau_m$.

[22]For instance, assuming quasi-linear utility, Boadway and Pestieau (2006) show in the context of a discrete-type optimal income tax model that the tax system will be more redistributive in the tagged group that has a higher proportion of high-ability persons. In a model with a continuum of types, also assuming quasi-linear utility, Cremer et al. (2010) show that if the skill distribution in one group first-order stochastically dominates the other, tagging calls for redistribution from the former to the latter group.

[23]This can be proved by starting from a situation $G_m = G_f$ and considering a small increase in G_f combined with a small reduction in G_m which yields a strict improvement in the social welfare function (assuming no wealth effects on labor supply).

The bracketed expressions reflect equity considerations. The first term in brackets in the numerator reflects that marginal tax rates for gender j should be higher the stronger is the (negative) correlation between the income of a single individual of gender j and its social marginal utility of income.

The second term reflects the (negative) correlation between the income of a married individual of gender j and the social marginal utility of income of the couple to which he/she belongs. For instance, if the income of the husband is more strongly correlated with the social marginal utility of income of the couple household than the income of the wife (in the sense that the social marginal utility of income for the couple declines more rapidly with male income) then this works in the direction of $\tau_m > \tau_f$. The importance of the equity term in the numerator has been emphasized by Apps and Rees (2011). Their argument is that it is a priori unclear whether the equity term contributes to the tax rate being lower or higher for females because this depends on which of the tax rates τ_m or τ_f is the more efficient instrument to redistribute income across couple households. In their model, the answer to this question depends purely on the matching process. In the present paper, intra-household bargaining implies that the social weight on the couple is given by ϑ_i^c. In particular this means that the social weight depends on the bargaining power parameter γ.

Finally, the third term reflects the intra-household externality associated with a tax-induced change in the outside options. It is the correlation between the income of singles of gender j and the value to society of the within-family reallocation which occurs due to a change in the tax rate on singles of gender j. When $V_{im} > V_{if}$ the concavity of the social welfare function implies that the term contributes positively to the tax rate on men and negatively to the tax rate on women. The externality term is large (in absolute terms) if $\mid V_{im} - V_{if} \mid$ is high when y_{ij}^s is high. For example, if the difference in utility between husband and wife is larger in high income households than in low income households, then a less progressive tax structure for women would be desirable as a way to improve the bargaining position of wives in high income households where inequality is relatively larger than in low income households.[24] Notice lastly that the externality terms in the formulas for τ_m and τ_f are multiplied by the factors $1 - \gamma$ and γ respectively. This means that when γ is high a lowering of the marginal tax rate for the woman is more important than an increase in the tax rate for the man. To understand this, note that as evident from equations (7) and (8) intra-household bargaining implies that each spouse in the couple first gets the utility they would obtain as single and then the remaining marital surplus is distributed unevenly according to the value of γ. An increase in τ_m impacts

[24]In the data, the inequality in wages between men and women rises with income.

the intra-household distribution because such a tax change reduces the utility the man would obtain as single and increases the marital surplus. Because this surplus is distributed between the spouses according to the value of γ, when γ is high, the husband obtains most of this surplus so the benefit of increasing the surplus is small. This is reflected by the $1-\gamma$ multiplying the externality term in the formula for τ_m.[25] Instead, when γ is high, because the woman always gets her single utility in the bargaining, a tax reduction for the female is worth more from the perspective of increasing the utility of married women.

Note the similarity between the externality terms described here and those which arise due to gender differentiated lump-sum transfers described in section 2.3 (and formally presented in expressions (39) and (40) in the appendix). The bargaining power of women can be strengthened either by differentiating the transfers G_m and G_f and/or differentiating the marginal tax rates τ_m and τ_f. The difference between these two instruments in affecting the intra-household distribution of resources is that differentiated marginal tax rates change the bargaining power differently in high income compared to low income households.

3.2 Couple-Based Taxation

Next I consider a tax system which is gender-neutral but distinguishes between singles and couples.

Proposition 3 *In the optimum the transfer G^c is determined by the condition*

$$\boldsymbol{E}[\vartheta^c] = 1 \tag{17}$$

G^s is determined according to the condition

$$\sum_{ij} \pi_{ij} \frac{W'(V_{ij}^s)}{\lambda} + \sum_i \pi_i \frac{(1-\eta)}{\eta} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left(\frac{1}{2} - \gamma\right) = 1 \tag{18}$$

or, alternatively, if $\gamma = 1/2$

$$E[\vartheta^s] = 1$$

where $\vartheta_i^s = \vartheta_{im}^s + \vartheta_{if}^s$.

Proof. See appendix D. ■

According to equations (17) and (18), if $\gamma = 1/2$, G^s and G^c should be set so that the average social marginal utility of income for both groups is equal to one. There are several reasons to differentiate lump-sum transfers between singles and couples.

[25] Similarly, γ is multiplying the externality term in the expression for τ_f.

First, lump-sum redistribution from couples to singles is one way to compensate singles for the fact that they do not benefit from having a spouse who contributes to the household public good.

Second, due to economies of scale in market consumption, couples are more efficient at generating utility which tends to increase the social marginal utility of income of couples (motivating a higher transfer directed towards couples). On the other hand, economies of scale imply that couples derive higher utility from a given amount of income which tends to decrease their social marginal utility of income (motivating a lower transfer to couples).

Third, if $\gamma \neq \frac{1}{2}$ and the spouse with the higher utility has more bargaining power, any lump-sum transfer to the couple will be distributed in a socially undesirable way (as reflected by the social marginal utility of income $\vartheta_i^c = z\left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma)\frac{W'(V_{if})}{\lambda}\right)$) which lowers the optimal transfer to the couple.

Finally, as mentioned in section 2.3, if $\gamma \neq \frac{1}{2}$ differential transfers between singles and couples affect the bargaining power within couple households as reflected by the second term in (18). Recall that the marital surplus was defined as the remaining resources in the family after each spouse has received the utility they would obtain as singles. As was evident from equations (7) and (8) this surplus is unevenly distributed between the spouses according to the value of γ. Thus, differentiated lump-sum transfers between singles and couples has intra-household distributional consequences. If $\gamma > 1/2$ and $V_{im}^c > V_{if}^c$, the concavity of the social welfare function implies that the externality term is guaranteed to be positive, which motivates higher lump-sum transfers to singles relative to couples.

Proposition 4 (Couple-Based Tax) *Under a couple-based tax scheme the optimal marginal tax rate for singles is*

$$\tau^s = \frac{1}{\overline{y}^{s\prime}} \sum_i \pi_i \Big[\sum_{j=m,f} (\vartheta_{ij}^s - 1) y_{ij}^s + \frac{(1-\eta)}{\eta} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left[(1-\gamma) y_{im}^s - \gamma y_{if}^s\right] \Big] \quad (19)$$

or, alternatively if $\gamma = 1/2$

$$\tau^s = \frac{\boldsymbol{cov}(\vartheta^s, y^s)}{\overline{y}^{s\prime}} + \frac{1}{\overline{y}^{s\prime}} \sum_i \pi_i \frac{(1-\eta)}{\eta} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \frac{1}{2} \left[y_{im}^s - y_{if}^s\right] \quad (20)$$

The optimal tax rate for couples is

$$\tau^c = \frac{\mathbf{cov}(\vartheta^c, y_m + y_f)}{\overline{y}^{c\prime}} \quad (21)$$

where $\vartheta_{ij}^s = \frac{W'(V_{ij}^s)}{\lambda}$ *and* $\vartheta_i^c = z\left(\gamma\frac{W'(V_{im})}{\lambda} + (1-\gamma)\frac{W'(V_{if})}{\lambda}\right)$ *are the social marginal utilities of singles and couples respectively and* $\overline{y}^{s'}$ $\overline{y}^{c'}$ *are the corresponding average (across gender and ability type) earned income responses to changes in the tax rate.*

Proof. See appendix D. ■

According to the proposition, the equity component of the marginal tax rate for singles, as reflected by the numerator of formula (19), should reflect the correlation between the income of singles and the social marginal utility of income of singles. There is also an externality term. In contrast to the analysis of lump-sum transfers above, the externality term does not vanish when $\gamma = 1/2$ (even though in this case the marital surplus is not affected by changes in τ_s). The reason for this is that τ_s applies to single men and women which in general do not have the same income. In couple i the bargaining power is determined by the utility levels U_{im}^s and U_{if}^s. These are influenced by y_{im}^s and y_{if}^s. If $y_{im}^s > y_{if}^s$, an increased tax rate on singles will increase the bargaining power of the wife, because the tax rate increase will cause a larger reduction in the outside option of the husband compared to the wife. If $\forall i$, $V_{im} > V_{if}$ (so that $\left[W'(V_{im}^c) - W'(V_{if}^c)\right]/\lambda < 0$) such an increase is welfare-improving. When $\gamma = 1/2$, the strength of this effect is increasing in the the distance $\mid y_{im}^s - y_{if}^s \mid$ and thus depends on the matching process.[26] Under positive assortative matching, considered here, the second term in (19) is guaranteed to be negative and since the denominator of (19) is negative as well, the average of these externality terms across all households contributes positively to the tax rate on singles.

If $\gamma \neq 1/2$, a change in τ_s changes the marital surplus which has additional distributional consequences. Recall from equation (8) that an increase in the tax rate on singles, reduces the utility of singles, and increases the amount of resources which is considered a surplus in the marriage (this is the part which is affected by γ). Thus, even if it is still true that $y_{im}^s > y_{if}^s$ so that an increase in τ^s has the overall effect of worsening the outside option of the husband relative to the outside option of the wife, an increase in τ^s increases the share of resources over which the husband has a higher control. Which of these effects dominates depends on the sign and contributions of the factors $\left[(1-\gamma)y_{im}^s - \gamma y_{if}^s\right]$ in (19) (given that $\left[W'(V_{im}^c) - W'(V_{if}^c)\right]/\lambda$ is negative so that the social welfare weight is higher for married women than for married men). If γ is sufficiently high, the externality term contributes negatively to the tax rate on singles.

Finally, the optimal tax rate on couples is given by formula (21) and reflects the correlation between total family income and the welfare weight on couple households. This formula is identical to that given in Boskin and Sheshinski (1983) and Apps and Rees (2011) for the case

[26] The matching process determines the wage rates in the couple (w_{im}, w_{if}). Because $w_{im}^s = w_{im}$ and $w_{if}^s = w_{if}$ the matching process influences the income distance $\mid y_{im}^s - y_{if}^s \mid$.

in which a uniform tax rate applies to the earned income of each spouse in a two-earner couple. The only difference is that the social weight $\vartheta_i^c = z\left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma)\frac{W'(V_{if})}{\lambda}\right)$ in (21) reflects how the marital surplus is divided in the couple household.

3.3 Full Optimum

Finally I consider the most sophisticated tax system.

Proposition 5 (Full Optimum, Marginal Tax Rates) *In the Full Optimum, the marginal tax rates for couples are*

$$\tau_f = \frac{\boldsymbol{cov}(\vartheta^c, y_f)}{\overline{y}_f'} \quad , \quad \tau_m = \frac{\boldsymbol{cov}(\vartheta^c, y_m)}{\overline{y}_m'}$$

and for singles

$$\tau_f^s = \frac{\boldsymbol{cov}(\widehat{\vartheta}_f^s, y_f^s)}{\overline{y}_f^{s\prime}} \quad , \quad \tau_m^s = \frac{\boldsymbol{cov}(\widehat{\vartheta}_m^s, y_m^s)}{\overline{y}_m^{s\prime}}$$

where $\widehat{\vartheta}_{if}^s = \frac{W'(V_{if}^s)}{\lambda} + \frac{1-\eta}{\eta}\gamma\frac{W'(V_{if})-W'(V_{im})}{\lambda}$ *and* $\widehat{\vartheta}_{im}^s = \frac{W'(V_{im}^s)}{\lambda} + \frac{1-\eta}{\eta}(1-\gamma)\frac{W'(V_{im})-W'(V_{if})}{\lambda}$ *are the (externality-corrected) net social marginal utilities of income for single women and men respectively. As before,* $\vartheta_i^c = z\left(\gamma\frac{W'(V_{im})}{\lambda} + (1-\gamma)\frac{W'(V_{if})}{\lambda}\right)$.

Proof. See appendix D. ■

In the proposition above, the expressions for τ_m and τ_f above have been written in a form which conforms with the model of Boskin and Sheshinski (1983) in which singles are absent. Although conceptually similar it is important to realize that in Proposition 5 the social marginal utility of couples depends on how resources are divided in the household. In contrast, the social marginal utility in Boskin and Sheshinski arises from the unitary model where the allocation within the household exactly conforms with the weights attached by the social planner.

The social marginal utility entering the expressions for τ_m^s and τ_f^s has two terms. The first is a standard term which would arise in a model with only singles. The second term reflects the externality of the tax treatment of singles on married couples. For instance, an increase in τ_m^s makes the tax system for single men more progressive; this means that high income single males face a relatively larger tax increase and therefore the wives of high income males experience a relatively larger boost in their bargaining power. This is efficient from a threat-point perspective if higher ability households also exhibit large inequality in terms of social marginal utility of income between husband and wife (as reflected by the difference $W'(V_{im}) - W'(V_{if})$). The reason this externality takes this particularly simple form is that a tax induced change in bargaining power of one spouse (though the single threat point) always implies a symmetric effect (but of opposite sign) for the other spouse.

Similar reasoning can be applied when I proceed to analyze optimal transfers because an externality term appears also in those formulas. Since it doesn't matter whether the husband or wife receives the transfer within marriage, one transfer is redundant and I set $G_m^c + G_f^c \equiv G^c$.

Proposition 6 (Full Optimum) *The transfer G^c and the transfers G_m^s and G_f^s satisfy, respectively*

$$\boldsymbol{E}[\vartheta^c] = 1 \quad , \quad \boldsymbol{E}[\widehat{\vartheta}_m^s] = \boldsymbol{E}[\vartheta_m^s] + \boldsymbol{E}[\xi_m^s] = 1 \quad \textit{and} \quad \boldsymbol{E}[\widehat{\vartheta}_f^s] = \boldsymbol{E}[\vartheta_f^s] + \boldsymbol{E}[\xi_f^s] = 1$$

where $\xi_{im}^s = \frac{1-\eta}{\eta}(1-\gamma)\frac{W'(V_{im})-W'(V_{if})}{\lambda}$ and $\xi_{if}^s = \frac{1-\eta}{\eta}\gamma\frac{W'(V_{if})-W'(V_{im})}{\lambda}$ are externality terms arising because an increase in the disposable income of singles has an external effect on marriages through the threat points.

Proof. See appendix D. ■

Giving an additional dollar of income to single women enables married women to claim an additional \$$\gamma$ from the husband (who loses \$$\gamma$) in the bargaining process. The social value of this within-couple reallocation of resources (in terms of government revenue) is precisely $\xi_{if}^s = \frac{1-\eta}{\eta}\gamma\frac{W'(V_{if})-W'(V_{im})}{\lambda}$ and its average value is $\mathbf{E}[\xi_f^s]$. This externality term is positive as long as the average social welfare weight on married women is higher than that of married men.

4 Quantitative analysis

The formulas presented in section 3 highlight in an intuitive manner the key considerations involved in the determination of optimal tax rates. However, using analytical expressions alone, little can be learned about the relative magnitudes of tax rates and the associated welfare gains of tagging. To derive further insights, I turn to numerical simulations of the linear tax model in the spirit of Stern (1976).

The first goal is to provide estimates of the welfare gains of employing tax systems (II)-(IV) relative to tax system (I). Second, I investigate the optimality of gender-based taxation and how singles should be taxed relative to couples. This is done using an empirically driven simulation approach where detailed wage data is obtained from population registers. To obtain measures of welfare gains in real economies it is necessary to study a model with income effects. This is the subject of the next subsection.

4.1 Model with income effects

Utility is now given by (dropping subscript i for clarity)

$$U_j = \frac{(c_j + H)^{1-\beta}}{1-\beta} - \chi\frac{1}{1+\phi}(\ell_j + h_j)^{1+\phi} \tag{22}$$

and for singles

$$U_j^s = \frac{(c_j^s + H_j^s)^{1-\beta}}{1-\beta} - \chi \frac{1}{1+\phi}(\ell_j^s + h_j^s)^{1+\phi}$$

where $\beta > 0$ is the Arrow-Pratt measure of relative risk-aversion, which here governs the rate at which marginal utility consumption decreases with income and hence controls the strength of the income effect. The calibration parameter χ measures the intensity of the disutility of working.[27] The utility function (22) is very standard and widely applied in the quantitative macroeconomics literature. Special cases of this function were also used in Saez (2001) and Weinzierl (2011) in their analysis of optimal taxes.

I focus on the decision-problem for couples. The Lagrangian is

$$\mathcal{L} = (U_m - U_m^s)^\gamma (U_f - U_f^s)^{1-\gamma} \\ + \mu \left[z((1-\tau_m)w_m\ell_m + (1-\tau_f)w_f\ell_f + G_m + G_f) - c_m - c_f \right]$$

where $H = \left[\frac{\omega_m}{\alpha^c}(h_m)^{\alpha^c} + \frac{\omega_f}{\alpha^c}(h_f)^{\alpha^c}\right]$. The first order conditions are given in appendix D.

Denote by $\Omega_m = \gamma(U_m - U_m^s)^{\gamma-1}(U_f - U_f^s)^{1-\gamma}$ and $\Omega_f = (1-\gamma)(U_m - U_m^s)^\gamma(U_f - U_f^s)^{-\gamma}$ the first derivatives of the Nash product with respect to U_m and U_f respectively. It is also useful to define $\Psi_f = \Omega_f/\Omega_m$ and $\Psi_m = 1/\Psi_f$ where

$$\Psi_f = \left(\frac{U_m - U_m^s}{U_f - U_f^s}\right) \Big/ \frac{\gamma}{1-\gamma} > 0. \tag{23}$$

The FOC:s for c_m and c_f can be combined into

$$\frac{(c_f + H)^{-\beta}}{(c_m + H)^{-\beta}} = \frac{\Omega_m}{\Omega_f} \quad \text{or, equivalently,} \quad \frac{c_f + H}{c_m + H} = \left[\Psi_f\right]^{\frac{1}{\beta}}. \tag{24}$$

Thus Ψ_f determines the ratio of female to male consumption. More specifically, the elasticity of substitution between female and male consumption w.r.t changes in Ψ_f is $1/\beta$. [28]

I interpret Ψ_f as describing the bargaining power of the female (with an analogous interpretation for Ψ_m). According to (23) it is defined as the ratio of the utility gains or 'marital surpluses' divided by the 'exogenous bargaining ratio' $\gamma/(1-\gamma)$. An equal division of resources (i.e. $\Psi_f = 1$) would obtain if these two quantities are equal. In the symmetric case $\gamma = 1/2$, $\Psi_f = (U_m - U_m^s)/(U_f - U_f^s)$. Intuitively, the spouse with the highest utility gain from marriage, will experience the largest drop in utility in the event of divorce, and thus will have lower bargaining power.

In the general case, an increase in the bargaining power of men, explicitly through an increase in γ, or implicitly through an increase in U_m^s, results in a lowering of Ψ_f which, according to

[27] To simplify the exposition I abstract from the parameter χ in the derivations below.

[28] That is, $\frac{1}{\beta} = d\log\left(\frac{c_f+H}{c_m+H}\right)/d\log\Psi_f$.

equation (24), results in a decrease of the share of consumption accruing to the wife. Furthermore, $\lim_{\gamma\to 1}\Psi_f = 0$.

The FOC:s for h_m and h_f can be written

$$(\ell_m + h_m)^{\phi} = [(c_m + H)^{-\beta} + \Psi_f (c_f + H)^{-\beta}]\omega^c h_m^{\alpha^c - 1} \tag{25}$$

and

$$(\ell_f + h_f)^{\phi} = [(c_f + H)^{-\beta} + \Psi_m (c_m + H)^{-\beta}]\omega^c h_f^{\alpha^c - 1} \tag{26}$$

The variable Ψ_f in (25) can be interpreted as the weight attached by the male to the marginal utility of the female when determining his contribution to the domestic public good in order to satisfy the bargaining solution.[29] A similar interpretation applies to Ψ_m in (26).

To simplify further note that $\Omega_j = \frac{\mu}{(c_j+H)^{-\beta}}$, hence re-arranging the FOC:s for h_m and h_f (see appendix) we obtain

$$\frac{(\ell_j + h_j)^{\phi}}{(c_j + H)^{-\beta}} = 2\omega^c h_j^{\alpha^c - 1}$$

or (using the FOC:s for ℓ_m and ℓ_f)

$$\frac{(\ell_j + h_j)^{\phi}}{(c_j + H)^{-\beta}} = z(1 - \tau_j)w_j$$

then

$$2\omega^c h_j^{\alpha^c - 1} = z(1 - \tau_j)w_j \tag{27}$$

and

$$h_j = \left(\frac{2\omega_j}{z(1 - \tau_j)w_j}\right)^{\frac{1}{1-\alpha^c}} \tag{28}$$

The bargaining process results in labor being efficiently allocated between household and market work. The labor allocation (in contrast to the division of consumption goods) is unaffected by the relative bargaining power of the spouses. Hence both spouses cooperate maximizing the total marital production or "marital surplus". To see this, note that household profits are maximized when the household allocation of labor is given by the solution to

$$\max_{h_m,h_f} \Pi = \max_{h_m,h_f} \left\{ 2\left(\frac{\omega_m}{\alpha^c}(h_m)^{\alpha^c} + \frac{\omega_f}{\alpha^c}(h_f)^{\alpha^c}\right) - (1 - \tau_m)zw_m h_m - (1 - \tau_f)zw_f h_f \right\}$$

[29] The presence of Ψ_f does not mean that the utility of the female enters the utility function of the male, but is simply a requirement of the cooperative solution.

The first order conditions to this problem coincide with (27).

Next I turn to the Hicksian elasticity of labor supply which is the key parameter in determining the efficiency cost of taxation. In appendix C I derive the compensated (Hicksian) elasticity of labor supply which is given by

$$\epsilon_j^k = \frac{1}{\phi + \beta\zeta_j^k} + \left(\frac{1}{\phi + \beta\zeta_j^k} + \frac{1}{1-\alpha^c}\right)\frac{h_j^k}{\ell_j^k} \tag{29}$$

where $k = s, c$ and $j = m, f$, $\zeta_j^c = \frac{z(1-\tau_j)w_j(\ell_j^c + h_j^c)}{(c_j + H)} < 1$ and $\zeta_j^s = \frac{(1-\tau_j^s)w_j(\ell_j^s + h_j^s)}{(c_j^s + H_j^s)} < 1$.

The first thing to note about (29) is that the elasticity is endogenous and depends on the ratio of household to market work. In a model without income effects, $\beta = 0$ and the above formula simplifies to $\frac{1}{\phi} + \left(\frac{1}{\phi} + \frac{1}{1-\alpha}\right)\frac{h_{ij}}{\ell_{ij}}$ which is the same as the formula for married couples given in Alesina et al. (2011) who show analytically, that if the market wages of women are lower than those of men, women engage relatively more in household work and have a higher labor supply elasticity. In the model studied here, because household productivity is a constant but market wage rates increase in the type of individuals, $\frac{h_{ij}}{\ell_{ij}}$ is decreasing in i.[30] Thus the model endogenously produces labor supply behavior consistent with both the empirical finding that the labor supply elasticity is high among those with low earnings, and that women generally exhibit a higher elasticity of labor supply than men. In addition, the product $\beta\zeta$ captures the income effect on labor supply. Recall the parameter β measures the rate at which the marginal utility of consumption decreases in response to shocks in non-labor income whereas the parameter ζ is the ratio of labor income to total income. Intuitively, the income effect is more important the higher is an individuals' fraction of wage income out of total income. Then, a marginal increase in the wage rate will have a large effect on the marginal utility of income.

4.2 Model with income effects and non-cooperative threat points

As is well known, the admissible efficient allocations of the bargaining model depends on the threat points. Up until now I have assumed that these are the utilities in case of divorce, i.e the utility of singles. This means that tax policy impacts the intra-household distribution purely through its effects on the utilities of singles. In particular the identity of the recipient of income within the couple household does not matter. How central is this mechanism for the results? To shed some light on this issue I now explore an alternative specification of threat points determined by the utility levels resulting from a simple form of strategic interaction within the couple. Since these threat points are internal to the couple, the spouse who controls the larger

[30] That this holds also in the presence of income effects is a consequence of the household good and market good being perfect substitutes.

fraction of family income will have the upper hand in the bargaining process.[31] To make the analysis interesting I focus on the specification with income effects. Declining marginal utility of consumption of the household good implies that there is an intra-household externality in the sense that contributions to the household public good of one spouse affects the marginal utility of consumption of the other spouse. I also assume that spouses are purely egoistic.[32]

I denote the new threat points by U_m^{NC} and U_f^{NC} which correspond to the utility levels produced by the non-cooperative game outlined below. The intra-household allocation is then defined as the solution to

$$\max_{\{c_{ij},\ell_{ij},h_{ij}\}_{j=m,f}} \Omega_i = (U_{im} - U_{im}^{NC})^{\gamma}(U_{if} - U_{if}^{NC})^{1-\gamma} \tag{30}$$

subject to a set of participation constraints (to be specified). Keep in mind that the solution to (30) is still efficient. The utility levels U_j^{NC} are determined by spouse j maximizing his/her own utility, taking the behavior of his/her spouse, denoted $-j$, as given:

$$U_{ij}^{NC} = \max_{\ell_{ij},h_{ij}} = \frac{1}{1-\beta}\left(z^{NC}\Big[(1-\tau_j)\ell_{ij}w_{ij} + G_j\Big] + H_i\right)^{1-\beta} - \frac{1}{1+\phi}(\ell_{ij}+h_{ij})^{1+\phi} \quad j = m,f \tag{31}$$

where $H_i = \left[\frac{\omega^c}{\alpha^c}(h_{ij})^{\alpha^c} + \frac{\omega^c}{\alpha^c}(h_{i[-j]}^{*})^{\alpha^c}\right]$.

A Cournot-Nash equilibrium is defined as the solution to the simultaneous set of equations

$$\ell_{ij}: \quad \left(z^{NC}\Big[(1-\tau_j)\ell_{ij}w_{ij} + G_j\Big] + H_i\right)^{-\beta} z^{NC}(1-\tau_j)w_{ij} = (\ell_{ij}+h_{ij})^{\phi}$$
$$h_{ij}: \quad \left(z^{NC}\Big[(1-\tau_j)\ell_{ij}w_{ij} + G_j\Big] + H_i\right)^{-\beta} \omega^c(h_{ij})^{\alpha^c-1} = (\ell_{ij}+h_{ij})^{\phi}$$

for $j = m,f$ where the equilibrium value of the household public good is $H_i = \left[\frac{\omega^c}{\alpha^c}(h_{im}^{*})^{\alpha^c} + \frac{\omega^c}{\alpha^c}(h_{if}^{*})^{\alpha^c}\right]$. The game reflects a situation where cooperation between spouses has broken down, but where there are reasons to stay inside marriage (such as when there are children in the household or when the financial and emotional costs of divorce are considered too large). The lack of cooperation implies that spouses cannot reap the full potential of household economies of scale, hence it is realistic to assume $z^{NC} < z$.

For the solution to (30) to be meaningful, the participation constraints $U_{ij} > U_{ij}^{NC}$ need to be satisfied. In addition I assume $U_{ij}^{NC} > U_{ij}^{s}, \forall i, j = m,f$ so that if cooperation breaks

[31] It should be noted, however, that cooperative and non-cooperative models will produce the same outcomes when the household production technology is separable and there are no externalities.

[32] This assumption serves mainly to sharpen the results. Degrees of 'caring preferences' can easily be introduced. In reality, most households probably lie somewhere between the fully cooperative and noncooperative solution. See Cherchye et al. (2011) for an interesting attempt to estimate the degree of caring inside households.

down, individuals prefer to remain in the couple compared to being single. This assumption is rationalized when there are costs associated with getting separated or divorced.

Combining the equilibrium conditions yields the following solution for household work

$$h_j^{NC} = \left(\frac{\omega^c}{z^{NC}(1-\tau_j)w_j} \right)^{\frac{1}{1-\alpha^c}}$$

upon comparison with (28), it is evident that h_j^{NC} is less than the efficient level of household work in bargaining couples. Without being able to coordinate household work, the household public good will be under-provided. It is interesting to compare the expression for h_j^{NC} with the corresponding expression for singles as well. Recall

$$h_j^s = \left(\frac{\omega^s}{(1-\tau_j^s)w_j} \right)^{\frac{1}{1-\alpha^s}}$$

If $\alpha^s = \alpha^c$ and $\omega_c < \omega_s$, then $h_j^{NC} < h_j^s$. In other words, members of non-cooperating couples work less at home than both singles and members of cooperating couples, for any given market ability level.

4.3 Data

The wage rates used in the numerical simulations are constructed from detailed wage distributions for men and women in ages 25-60 working at least part-time in Sweden in 2005.[33] Data has been combined from three sources, 'Flergenerationsregistret', 'Louise-databasen' and 'Lönestrukturstatistiken'. These statistics cover men and women working in the private and public sector. The original data set includes 1 457 931 wages for women and 1 519 921 wages for men. For numerical tractability and without affecting the qualitative insights, in the numerical simulations each distribution is represented by 50 taxpayer types.[34]

4.4 Calibration

In order to perform numerical simulations, the values of all parameters must be specified. Some of these parameter values are chosen from available empirical estimates and some are calibrated to match empirical targets. In accordance with Einav et al. (2010) and their assessment of the appropriate benchmark value of β I set the parameter governing the curvature of consumption of

[33]The gender wage-gap is not constant across ages, hence in reality it might be desirable to make a gender-based tax age-dependent.

[34]The wage distributions are approximated using quantiles. The person with the lowest productivity is represented by the 2:nd percentile wage rate, the second lowest by the 4:th and so forth up until the 98:th percentile. The highest productivity type is associated with the wage rate corresponding to the 99:th percentile. This gives a total of 50 taxpayer types.

household and market goods equal to 3 (e.g. $\beta = 3$).[35] Following Saez (2001) I set the disutility of labor parameter ϕ to 2. This is a standard value often used in the optimal tax literature. A key reason for taxing couples differently than singles are economies of scale hence it is important to be explicit about these parameter choices. Browning et al. (2004) develop 'indifference scales' for estimating consumption economies of scale. They answer the question "How much income would an individual living alone need to attain the same indifference curve over goods that the individual attains as a member of the household?". Lewbel and Pendakur (2008) adopt this approach and estimate that a single female would need approximately two-thirds the couple's income to reach the same indifference curve.[36] Approximating their estimate in the current model yields $z = 1.33$ (details are given in appendix E). With the price and economies of scale of the market good fixed, the parameters in the household production technology determine the ratio of household consumption to market goods. I calibrate the parameters of the production function to result in empirically relevant ratios of h_j/ℓ_j and h_j^s/ℓ_j^s, $j = m, f$.[37]

I obtain rough estimates of these quantities from the 2009 American Time Use Survey published by the US Bureau of Labor Statistics. Finally, I adopt a revenue requirement equal to 10% of equilibrium output. In the benchmark, the exogenous bargaining power parameter is $\gamma = 1/2$. The social welfare function is a weighted sum of individual utilities where the weight on type i is ρ^i defined as $\rho^i = 1/i^x$ where $x \geq 0$ represents the government's inequality aversion.[38] For $x = 0$ I obtain the unweighted utilitarian social welfare function and the max-min social welfare function is obtained as $x \to \infty$. In the benchmark case I assume mildly redistributive preferences given by $x = 0.25$. I choose this value of x because it generates, in the benchmark optimum, a degree of progressivity in the tax structure which resembles actual tax systems. Higher values of x makes the tax system more progressive, with higher marginal tax rates and larger transfers.

To construct an equivalent-variation type of welfare gain measure of policy reform I proceed as follows. I calculate the minimum amount of extra revenue Λ which needs to be injected into the government budget constraint in the pre-reform equilibrium, to reach the social welfare

[35] These authors note that a substantial literature uses a baseline value of 3 for this coefficient (Hubbard, Skinner, and Zeldes, 1995; Engen, Gale, and Uccello, 1999; Mitchell et al., 1999; Scholz, Seshadri, and Khitatrakun, 2003; and Davis, Kubler, and Willen, 2006). The reader is referred to the references in Einav et al. (2010).

[36] As pointed out by the authors, this value value is an upper bound as the bundle of goods an individual would obtain when single and as a member of couple household is not the same.

[37] The parameter choices are: $\alpha^c = \alpha^s = 0.6$, $\omega^c = \overline{\omega}$, $\omega^s = 0.7\overline{\omega}$ where $\overline{\omega} = w_{25}/2$ and w_{25} is the 25:th percentile of the wage distribution for men.

[38] Formally, the objective is expressed as $\sum_{ijk} \pi_{ij}^k \rho^i V_{ij}^k$ where ρ^i is the Pareto weight on an individual of type (i, j, k). Note that the same weight ρ^i applies to all individuals of type i, regardless of gender (j) and marital status (k).

level of the post-reform equilibrium. Finally Λ is divided by aggregate income in the pre-reform economy to obtain a welfare gain measure expressed in terms of percentage points of GDP, which is also the quantity reported in table 1 for the different tax optima. The fraction of singles is set to 1/2.[39]

5 Results

The model was set up in the software AMPL and solved using the latest version of the state-of-the-art nonlinear optimization package KNITRO (version 8.0).

First I assess the empirical validity of the model. Table 5 (on page 40 in the appendix) presents some key statistics. The top panel shows the ratio of household work to market work. Consistent with time-use data, men work more in the market relative to home work and couples work more at home than singles.[40] In Table 6 (on page 40) it can be seen that even though the gender wage-gap implies that women work more at home and men work more in the market, the gender differences in the amount of total work is much smaller. This is consistent with empirical evidence in Burda et al. (2007) and the assumption that market work and household work are perfect substitutes in the utility function.

Figures 1 to 6 show the distribution of consumption of private and household goods, market work and household work across ability types for the four different tax systems. As evident from figures 1 and 2 , gender inequality in consumption is increasing in the wage rate for both singles and couples.[41] The gender based policies close this gap slightly. Consistent with the assumed household technology, couples substitute market consumption for household consumption as the wage rate increases. This can also be seen from figures 5 and 6 which show declining profiles of household work.

The results for the benchmark parameterization are presented in table 1 where the marginal

[39]According to the U.S. Census Bureau, around 57% of the population reported in 2000 that they were living with a partner (Reference: Hobbs, F. (2005) "Examining American Household Composition: 1990 and 2000: Census 2000 Special Reports"). According to estimates in "Hushållens ekonomi (HEK)" published by Statistics Sweden, out of a sample of 4 628 000 there were 1 547 000 cohabiting couples (with and without children) in ages 18-64 and 1 478 000 singles in 2009. Hence as a rough estimate I choose $\eta = 1/2$.

[40]The figures for single and married men replicate the corresponding moments in the 2009 US Time-use survey closely. I have estimated the corresponding numbers for the ratios h_m/ℓ_m and h_m^s/ℓ_m^s from the data and found that they are 0.326 and 0.288 respectively. In the model they are 0.3231 and 0.2330, respectively. For women the fraction of time spent in household work in the data is somewhat higher than in the simulations. A possible reason for this mismatch is that the gender-gap in wages is higher in the US than in Sweden. A closer match could, at least in principle, be achieved by introducing gender-specific household technologies.

[41]This is due to the fact that the gender wage-gap is increasing with ability type.

tax rates and transfers for the four different tax systems are reported.[42] To get a sense of magnitude of the size of the lump-sum component of the income tax schedules one can refer to the bottom panel of table 1 which expresses the lump-sum transfers as a fraction of average consumption.

The first thing to note about table 1 is that, consistent with earlier findings in the literature, Gender-Based Taxation (GBT) involves a (substantially) higher tax rate on male labor supply. The lump-sum transfer is also much lower for male agents. Looking next at the CBT column, it is clear that CBT implies taxing singles at a much higher marginal rate than couples. Singles also receive a generous lump-sum transfer while couples face a small lump-sum tax. This implies redistribution from couple to single households. That couples should receive lower lump-sum transfers in the optimum is a very robust finding and holds over a broad range of parameter specifications. This can be explained by the presence of economies of scale in couple households and the fact that couples work more in the untaxed (household) sector than singles. The pattern of marginal tax rates in the CBT is also consistent with the intuition given in section 3.2; higher tax rates on singles is optimal from a within-household redistribution perspective as men have higher potential income as singles.

The Full Optimum more or less combines the structure of taxes of the GBT and CBT optimum. In line with this interpretation, the welfare gains of the Full Optimum roughly equals the sum of the welfare gains of GBT and CBT combined. In the Full Optimum there is substantial difference between the tax rates on single men and single women. This is coherent with the idea that this is efficient from a threat-point perspective (because the utility of single individuals influences the division of consumption within the couple). Furthermore, lump-sum transfers are much less differentiated between single men and women in the Full Optimum as compared to the GBT optimum. This suggests that differentiated marginal tax are preferred to differentiated lump-sum transfers as an instrument to promote within-household equity.

The welfare gains of GBT, CBT and the Full Optimum are equal to 0.74%, 1.31% and 1.93%, respectively, of aggregate consumption. Next, I investigate how these welfare gains are affected by changes in the exogenous bargaining power parameter γ and the magnitude of the gender wage-gap.

By increasing all male wage rates by 5% I have tried to mimic the effects of increasing

[42] Note that in the case with divorce threat-points, except in the case where taxes explicitly depend on marriage-status, the demogrant G on couples is always restricted to be the sum of the (single) male and female demogrants. This is because with external threat points, the identity of the individual in the couple receiving the transfer is irrelevant for the intra-household distribution.

the gender wage-gap.[43] This is a simple yet crude way of increasing the wage-gap because it increases the wage-gap with the same percentage at each percentile of the wage distribution. The results of this exercise are presented in table 3. The welfare are now 2.44%, 2.73% and 3.64%, respectively. This substantial increase in the welfare gains can be explained by noting that the wage gap is smallest at the bottom of the income distribution and largest at the top. Thus increasing all male wages by a constant factor tends to increase gender-inequality relatively more in households carrying a large weight in the social welfare function (given that the sequence of social welfare weights is decreasing in the skill type of agents). Nonetheless, it is a worthy exercise as it highlights the importance of the gender wage-gap for the achievable welfare gains. The qualitative pattern of the income tax systems remain, however, the same.

Next, I consider an increase in the exogenous bargaining power of men from $\gamma = 0.5$ to $\gamma = 0.75$. This is an interesting exercise as it highlights the effect of a very uneven bargaining power on optimal taxes and household allocations. The results are presented in table 2.

The first thing to note is that the result that the male tax rate should be higher than the female tax rate is reversed. [44] The effect of the change in γ is that men appropriate a larger share of consumption and work less in the market and at home (although the fraction of time spent in household work compared to market work is still higher for women). Irrespective of the value of γ, it is true that women have a higher elasticity of labor supply. Thus efficiency principles make it still desirable to strengthen the incentives for women to work (suggesting $\tau_f < \tau_m$). However, when γ is large, the gain in efficiency is small relative to the benefit of reducing within-household inequality by affecting the threat points through an appropriate adjustment of gender-based transfers. This is because additional family income generated by an increase in the labor supply of married women will be very unevenly distributed when γ is high. Instead, GBT improves the distribution of consumption within households (through the externality effect due to the higher utility of single women) by offering a much larger lump-sum transfer to women relative to men when γ is high as compared to when γ is low. Given such large transfers to women it is intuitive that the optimal phasing out rate should be higher for women, making $\tau_f > \tau_m$ a reasonable result in light of the planner's aversion to within-group inequality.

The second thing to note from the change from $\gamma = 0.5$ to $\gamma = 0.75$ is that the welfare gain of CBT goes down significantly (and is now 0.44%) and that the welfare gain of gender-based taxation slightly goes up to 0.81%. A partial explanation for the drop in the welfare gain of

[43] This can alternatively be interpreted as a rough way of increasing the degree of assortative matching.

[44] This reversal of the GBT result also holds true in the model with non-cooperative threat points as γ is increased.

CBT is that raising γ from 0.5 to 0.75 magnifies the importance of within-household inequality as compared to other sources of inequality in the economy. The drop in the welfare gain of CBT can also be explained by recalling the discussion following Proposition 4 (on page 18). There two effects were highlighted. Whenever $y^s_{im} > y^s_{if}$ an increase in the tax rate on singles will reduce the reservation utility of the husband by more than it will reduce the reservation utility of the wife (which is beneficial from a within-household perspective). However it was also highlighted that an increased tax rate on singles increases the marital surplus. When $\gamma = 0.75$ the husband appropriates a much larger share of this surplus than the wife, hence it is unlikely to be optimal to increase the surplus as a way to act on intra-household inequality. This explains why, in the CBT column in table 2, the tax rate on singles is much lower compared to the case when $\gamma = 0.5$ and also explains the drop in the welfare gains associated with CBT.

Noncoopearative (internal) threat-points

In table 4 the results for the case with noncooperative threat points is displayed. The parameter choices are the same as before except that I have now imposed the additional parameter $z_{NC} = 1 < z$.[45] The first thing to note is that the structure of an optimal gender-based tax remains remarkably similar to the benchmark model. However the welfare gain is much smaller (0.28% compared to 0.74%). The differences between the two models are revealed in the Full Optimum. In the Full Optimum it is evident that the optimal tax system with internal threat points features, compared to the benchmark and in accordance with intuition, a narrowing of the marginal tax differential between single men and women accompanied with a slightly larger marginal tax differentiation between married men and women. Furthermore, the differential between the lump-sum transfers for men and women is much higher for married couples than for singles. These findings are consistent with the fact that when the threat-points are internal to marriage, a strengthening of the bargaining power of married women is achieved through a differentiation in the tax treatment applying to *married* men and women. In sum, the introduction of the alternative threat points does not affect the conclusions regarding the structure of gender-based taxation but does matter for a tax system differentiating between singles and couples.

[45] This reflects that households which are not cooperating cannot reap the benefits of economies of scale in market consumption. However, non-cooperating couples still benefit from the household public good to which they voluntarily contribute. Remember that it is only the threat points which are determined by the inefficient equilibria. The allocation within the couple is still fully efficient.

Table 1: Benchmark ($\gamma = 0.50$)

Instrument	Benchmark	GBT	CBT[1]	Full Optimum[1]
τ_m	30.13 %	25.99 %	16.66 %	25.07 %
τ_f	30.13 %	17.29 %	16.66 %	15.70 %
τ_m^s	30.13 %	25.99 %	30.79 %	32.70 %
τ_f^s	30.13 %	17.29 %	30.79 %	18.53 %
G_m	0.8728	0.4423	-0.1022	-0.0205
G_f	0.8728	0.7014	-0.1022	-0.0205
G_m^s	0.8728	0.4423	1.3568	1.2048
G_f^s	0.8728	0.7014	1.3568	1.3652
$G_m/E[c_m]$	0.1748	0.0886	-0.0205	-0.0041
$G_f/E[c_f]$	0.1729	0.1390	-0.0202	-0.0041
$G_m^s/E[c_m^s]$	0.1848	0.0937	0.2874	0.2552
$G_f^s/E[c_f^s]$	0.1760	0.1414	0.2736	0.2753
Welfare Gain		0.74 %	1.31 %	1.93 %

[1] Only the sum $G_m + G_f$ is determined by the model.

Table 2: Benchmark ($\gamma = 0.75$)

Instrument	Benchmark	GBT	CBT[1]	Full Optimum[1]
τ_m	22.94 %	18.63 %	22.01 %	9.25 %
τ_f	22.94 %	27.50 %	22.01 %	30.89 %
τ_m^s	22.94 %	18.63 %	26.60 %	28.89 %
τ_f^s	22.94 %	27.50 %	26.60 %	20.41 %
G_m	0.6082	-0.0395	0.4582	0.3970
G_f	0.6082	1.2272	0.4582	0.3970
G_m^s	0.6082	-0.0395	0.8794	0.5605
G_f^s	0.6082	1.2272	0.8794	0.9887
$G_m/E[c_m]$	0.0960	-0.0062	0.0723	0.0627
$G_f/E[c_f]$	0.1427	0.2878	0.1075	0.0931
$G_m^s/E[c_m^s]$	0.1313	-0.0085	0.1898	0.1210
$G_f^s/E[c_f^s]$	0.1302	0.2627	0.1882	0.2116
Welfare Gain		0.81 %	0.44 %	1.23 %

[1] Only the sum $G_m + G_f$ is determined by the model.

Table 3: Benchmark, 5% increased gender wage-gap, ($\gamma = 0.5$)

Instrument	Benchmark	GBT	CBT	Full Optimum
τ_m	37.66 %	26.64 %	15.82 %	25.43 %
τ_f	37.66 %	17.19 %	15.82 %	15.31 %
τ_m^s	37.66 %	26.64 %	32.58 %	34.10 %
τ_f^s	37.66 %	17.19 %	32.58 %	18.21 %
G_m	1.1398	0.3655	-0.1047	0.0343
G_f	1.1398	0.8776	-0.1047	0.0343
G_m^s	1.1398	0.3655	1.4425	1.1648
G_f^s	1.1398	0.8776	1.4425	1.5136
$G_m/E[c_m]$	0.2209	0.0709	-0.0203	0.0066
$G_f/E[c_f]$	0.2184	0.1681	-0.0201	0.0066
$G_m^s/E[c_m^s]$	0.2367	0.0759	0.2995	0.2419
$G_f^s/E[c_f^s]$	0.2256	0.1737	0.2855	0.2996
Welfare Gain		2.44 %	2.73 %	3.64 %

Table 4: Noncooperative threat points ($\gamma = 0.5$)

Instrument	Benchmark	GBT	CBT	Full Optimum
τ_m	22.09 %	26.00 %	18.68 %	25.15 %
τ_f	22.09 %	17.27 %	18.68 %	14.67 %
τ_m^s	22.09 %	26.00 %	27.00 %	28.43 %
τ_f^s	22.09 %	17.27 %	27.00 %	22.73 %
G_m	0.5703	0.4435	0.2055	0.1384
G_f	0.5703	0.6996	0.2055	0.3524
G_m^s	0.5703	0.4435	1.0071	0.8084
G_f^s	0.5703	0.6996	1.0071	1.1827
$G_m/E[c_m]$	0.1097	0.0853	0.0395	0.0266
$G_f/E[c_f]$	0.1091	0.1338	0.0393	0.0674
$G_m^s/E[c_m^s]$	0.1193	0.0928	0.2107	0.1691
$G_f^s/E[c_f^s]$	0.1227	0.1505	0.2167	0.2544
Welfare Gain		0.28 %	0.65 %	1.02 %

6 Concluding remarks

A first purpose of this paper has been to investigate the optimality of gender-based taxes in a setting allowing both for intra-household and inter-household inequality. The vehicle of analysis has mainly been a quantitative simulation model calibrated to agree with empirical facts. In my baseline model I find support for gender-based taxation with higher tax rates on men than for women. Somewhat surprisingly, this result is reversed when the bargaining power of men is high (although *lump-transfers* should still be higher for women than for men). If economies of scale are important and household goods have a public good character, optimal linear taxation implies that there should be significant redistribution of income from couples to singles. The results above are robust to perturbations in the modeling framework. Specifically I have obtained qualitatively similar results when the bargaining threat-points are given by the utilities of single individuals and when these points are given by the solution to a simple non-cooperative game within the household. The welfare gains of gender-based taxation depend primarily on the size of the gender-wage gap and the distribution of bargaining power in the household. When the gender wage-gap is large and the bargaining power is skewed in favor of the husband, the welfare gains of gender-based taxation are likely to exceed one percent of aggregate income. This suggests that gender-based taxation could be a useful instrument to raise social welfare in developing countries with high gender inequality.

While a tax on gender might seem controversial, it does not require a literal interpretation. It can instead be interpreted as employing selective taxation measures for secondary earners or just subsidizing day care expenditures. These policies effectively increase the net-of-tax wage rate for women. In practice, discriminating taxes on the basis of gender might not be politically feasible. But as women often are secondary earners in families, an optimal income tax should therefore differentiate marginal tax rates between primary and secondary earners. Indeed, many countries do have some form of selective taxation where the marginal tax rate on the secondary earner is different from the marginal tax rate on the primary earner. In fact this is an implicit feature of all progressive tax schedules.[46]

A limitation of the analysis is that the decision to marry or stay single is exogenous. However, from the perspective of tax policy, the distortions to the marriage are second order compared to the potentially sizable first order distributional gains of taxing singles and couples differently.[47]

[46]In countries where the effective marginal tax rate contains a social security payroll tax component, if evaluated net of the present actuarial value of future retirement benefits, marginal tax rates are lower for women on average because of their longer life expectancy.

[47]Anderberg (2008) surveys the literature and reports that it has found only modest effects of taxes on marriage. Gruber (2004) studies the effect of relaxing divorce law on children's outcomes and argues convincingly that these

Perhaps most importantly I have abstracted from what is commonly labeled as "family needs" arising due to the presence of children in the households. These are in the political discourse usually given as arguments for providing positive transfers to families. The results in this paper add to this discussion and can be interpreted as weakening the argument for providing larger transfer payments to couples relative to singles (without children).

Finally, in this paper I have assumed that couples Nash-bargain over the intra-household allocation. In future work it would be interesting to consider other models of intra-family decision-making.

effects are more likely to have been obtained through a change in the behavior of families that remain intact rather than through increased incidence of divorce. The current model can be rationalized with an economy where there are people who simply prefer to stay single, are unable to find a spouse (due to high search costs) or simply are unable to cooperate successfully within a couple household. Similarly, couples can be thought of as having made sunk, relationship-specific investments (such as emotional investments) or made joint financial investments in housing.

References

Alesina, A., Ichino, A., and Karabarbounis, L. (2011). Gender-based taxation and the division of family chores. *American Economic Journal: Economic Policy*, 3(2):1–40.

Alm, J., Whittington, L. A., and Fletcher, J. (2002). Is there a 'singles tax'? the relative income tax treatment of single households. *Public Budgeting and Finance*, 22(2):69–86.

Anderberg, D. (2008). Tax credits income support, and couple decisions. *International Tax and Public Finance*, 15:499–526.

Apps, P. F. and Rees, R. (1997). Collective labor supply and household production. *Journal of Political Economy*, 105(1):pp. 178–190.

Apps, P. F. and Rees, R. (2011). Optimal taxation and tax reform for two-earner households. *CESifo Economic Studies*, 57:283–304.

Atkinson, A. B. and Stiglitz, J. (1980). *Lectures in Public Economics*. McGraw-Hill, New York.

Bastani, S., Blomquist, S., and Micheletto, L. (2010a). Public provision of private goods, tagging and optimal income taxation with heterogeneity in needs. UCFS Working Paper 14, Uppsala Center for Fiscal Studies.

Bastani, S., Blomquist, S., and Micheletto, L. (2010b). The welfare gains of age related optimal income taxation. CESifo Working Paper 3225, CESifo.

Becker, G. S. (1965). A theory of the allocation of time. *Economic Journal*, 75(299):493–517.

Blomquist, S. and Micheletto, L. (2008). Age-related optimal income taxation. *The Scandinavian Journal of Economics*, 110(1):pp. 45–71.

Blundell, R. and MaCurdy, T. (1999). Labor supply: A review of alternative approaches. In Ashenfelter, O. and Card, D., editors, *Handbook of Labor Economics*. Elsevier, Amsterdam.

Blundell, R. and Shephard, A. (2011). Employment, hours of work and the optimal taxation of low income families. *Review of Economic Studies (forthcoming)*.

Boadway, R. and Pestieau, P. (2006). Tagging and redistributive taxation. *Annales d'Economie et de Statistique*, 83/84:123–147.

Boskin, M. J. and Sheshinski, E. (1983). Optimal tax treatment of the family: Married couples. *Journal of Public Economics*, 20:281–97.

Browning, M., Chiappori, P. A., and Lewbel, A. (2004). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. Working Paper 588, Boston College.

Burda, M., Hamermesh, D. S., and Weil, P. (2007). Total work, gender and social norms. IZA Discussion Paper 2705, Institute for the Study of Labor (IZA).

Cherchye, L., Muynck, T. D., and DeRock, B. (2011). Noncooperative household consumption with caring. Working Paper, Tillburg Univeristy.

Chiappori, P.-A. D. (2009). Non-unitary models of household behavior: a survey of the literature. IZA Discussion Paper 4603, Institute for the Study of Labor (IZA).

Cremer, H., Gahvari, F., and Lozachmeur, J. M. (2010). Tagging and income taxation: theory and an application. *American Economic Journal: Economic Policy*, 2(1):31–50.

Einav, L., Finkelstein, A., and Schrimpf, P. (2010). Optimal mandates and the welfare cost of asymmetric information: Evidence from the u.k. annuity market. *Econometrica*, 78:1031–1092.

Gronau, R. (1977). Leisure, home production and work - the theory of the allocation of time revisited. *The Journal of Political Economy*, 85:1099–1123.

Gruber, J. (2004). Is making divorce easier bad for children? the long?run implications of unilateral divorce. *Journal of Labor Economics*, 22(4):pp. 799–833.

Immonen, R., Kanbur, R., Keen, M., and Tuomala, M. (1998). Tagging and taxing: The use of categorical and income information in designing tax/transfer schemes. *Economica*, 65:179–192.

Lewbel, A. and Pendakur, K. (2008). Estimation of collective household models with engel curves. *Journal of Econometrics*, 147(2):350–358.

Lise, J. and Seitz, S. (2011). Consumption inequality and intra-household allocations. *Review of Economic Studies*, 78:328–355.

Lundberg, S. J. and Pollak, R. A. (1993). Separate spheres bargaining and the marriage market. *Journal of Political Economy*, 101:988–1010.

Lundberg, S. J. and Pollak, R. A. (1996). Bargaining and distribution in marriage. *Journal of Economic Perspectives*, 10:139–58.

Mankiw, N. G. and Weinzierl, M. (2010). The optimal taxation of height: A case study of utilitarian income redistribution. *American Economic Journal: Economic Policy*, 2(1):155–76.

Manser, M. and Brown, M. (1980). Marriage and household decision-making: A bargaining analysis. *International Economic Review*, 21:31–44.

McElroy, M. B. and Horney, M. J. (1981). Nash-bargained household decisions: toward a generalization of the theory of demand. *International Economic Review*, 22:333–349.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies*, 38:175–208.

Ooghe, E. and Peichl, A. (2010). Fair and efficient taxation under partial control: theory and evidence. CESifo Working Paper 3518, CESifo.

Rosen, H. S. (1977). Is it time to abandon joint filing? *National Tax Journal*, 30(4):423–428.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies*, 68:205–229.

Sheshinski, E. (1972). The optimal linear income tax. *Review of Economic Studies*, 39:297–302.

Stern, N. H. (1976). On the specification of models of optimum income taxation. *Journal of Public Economics*, 6:123–162.

Viard, A. D. (2001). Optimal categorical transfer payments: The welfare economics of limited lump-sum redistribution. *Journal of Public Economic Theory*, 3(4):483–500.

Weinzierl, M. C. (2011). The surprising power of age-dependent taxes. *Review of Economic Studies*, 78(4):1490–1518.

A Calibration graphs and tables

Table 5: Some Key Statistics (Benchmark)

	Benchmark	GBT	CBT	FO
h_m/l_m	0.3231	0.2696	0.1865	0.2352
h_f/l_f	0.4192	0.2674	0.2402	0.2331
h_m^s/l_m^s	0.2330	0.1945	0.2586	0.2672
h_f^s/l_f^s	0.2845	0.1990	0.3178	0.2265
H/c_m	0.6338	0.5565	0.5347	0.5947
H/c_f	0.6529	0.5455	0.5498	0.5778
H_m^s/c_m^s	0.4481	0.4299	0.4269	0.4558
H_f^s/c_f^s	0.5366	0.4139	0.5086	0.3923
H_m^s/H	0.6573	0.6829	0.7440	0.7352
H_f^s/H	0.7232	0.6977	0.8185	0.7122

Table 6: Consumption and Labor Supply (Benchmark)[1]

	Benchmark	GBT	CBT	FO
		Difference Male vs. Female		
c	0.0361	-0.0092	0.0338	-0.0180
ℓ	0.1084	0.0238	0.0810	0.0277
h	-0.2389	-0.0536	-0.2389	-0.0426
$\ell + h$	0.0356	0.0216	0.0361	0.0265
		Difference Couple vs. Single		
c	0.1088	0.1114	0.1149	0.0199
ℓ	-0.2389	-0.2052	0.0488	0.0029
h	0.0855	0.0855	-0.2241	-0.0329
$\ell + h$	-0.1611	-0.1500	-0.0083	-0.0062

[1] Entries are expressed as difference in log means and can be interpreted as percentage differences.

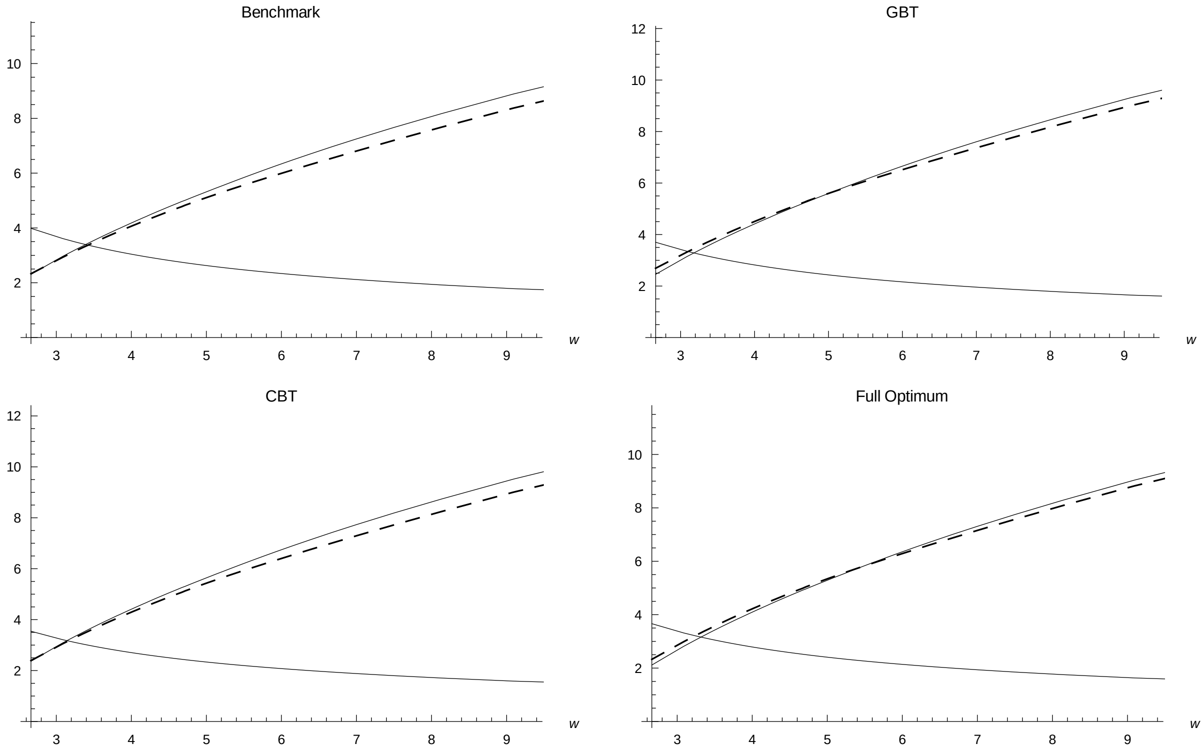

Figure 1: Consumption Married (solid line = males, dashed = females). The downwards sloping line is the household public good.

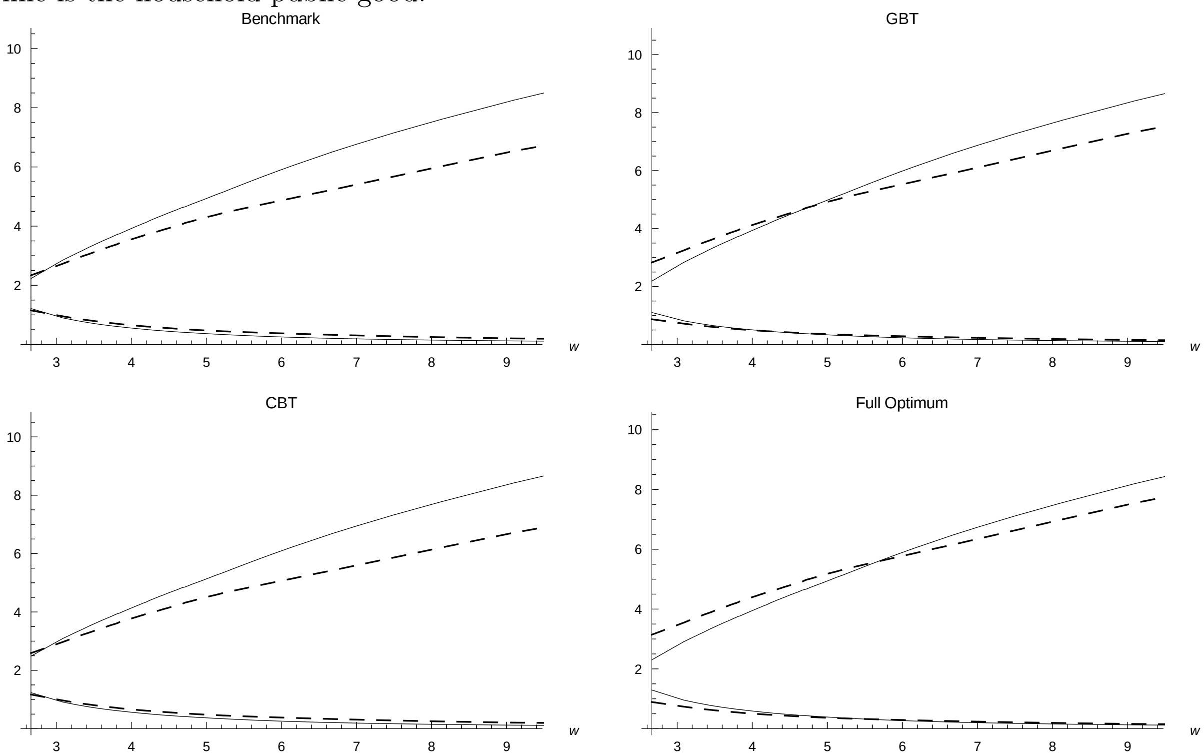

Figure 2: Consumption Singles (solid line = males, dashed = females). The downwards sloping lines are the household consumption.

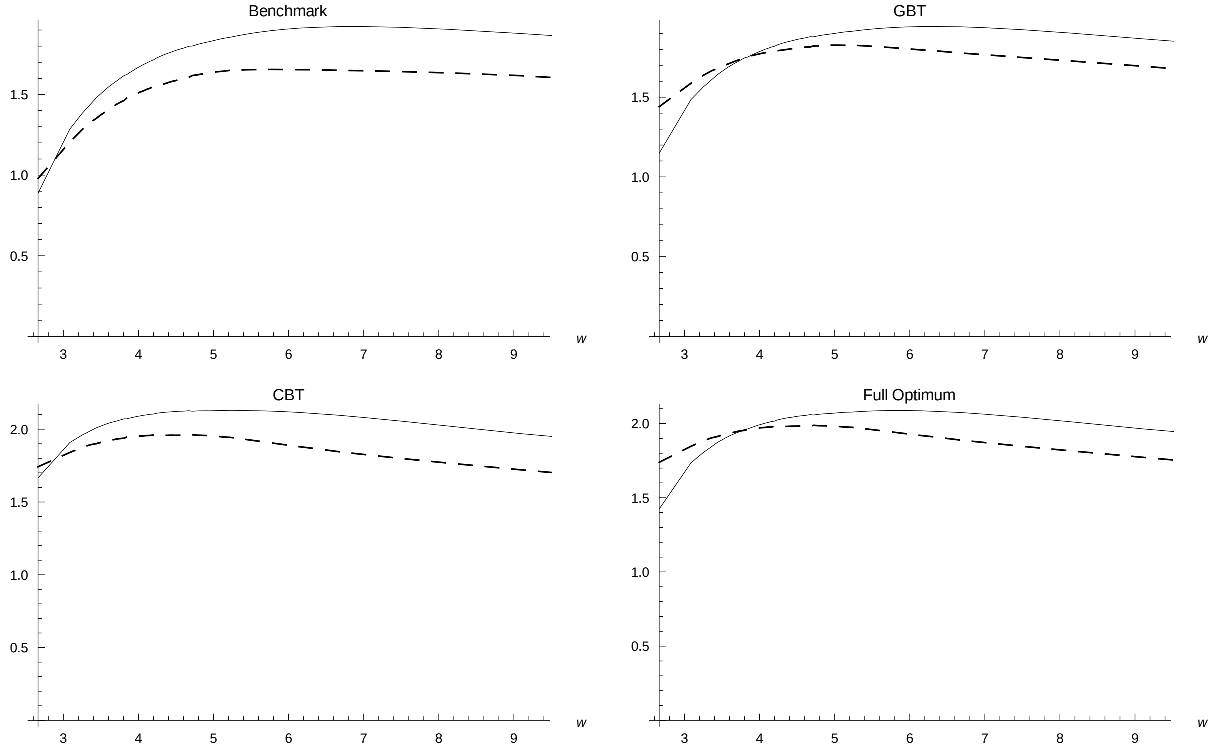

Figure 3: Labor Supply Married (solid line = males, dashed = females)

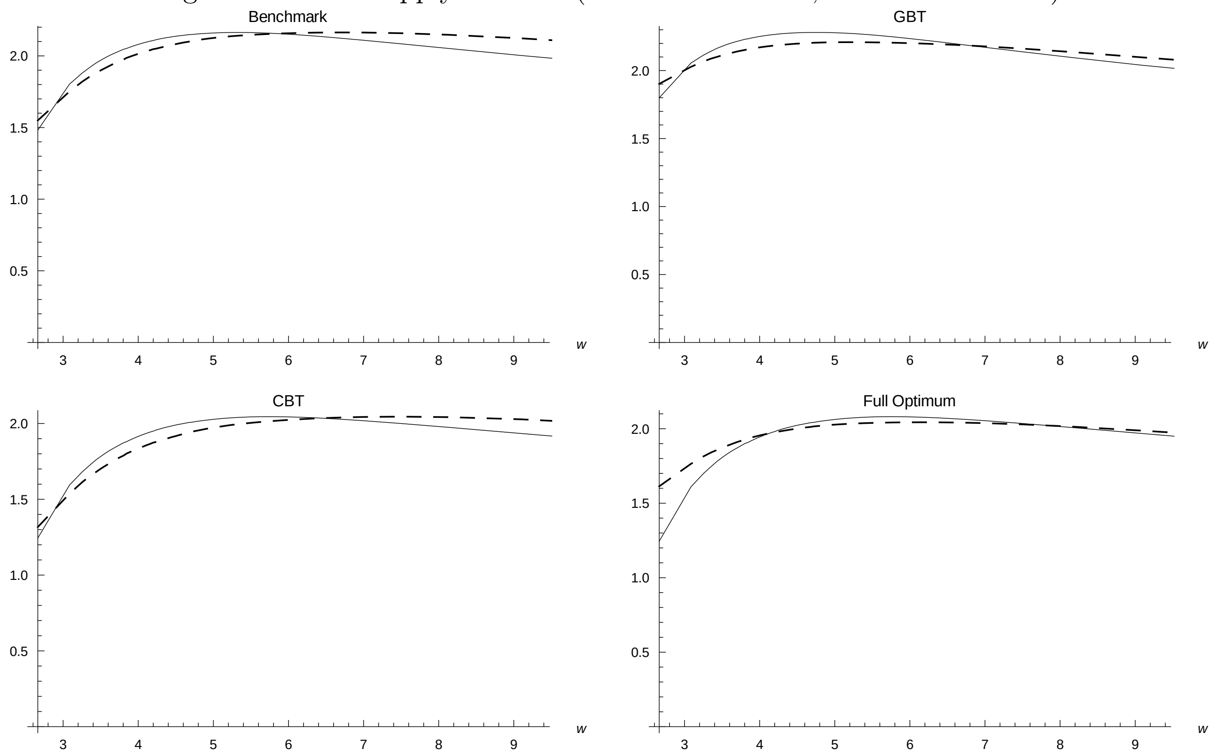

Figure 4: Labor Supply Singles (solid line = males, dashed = females)

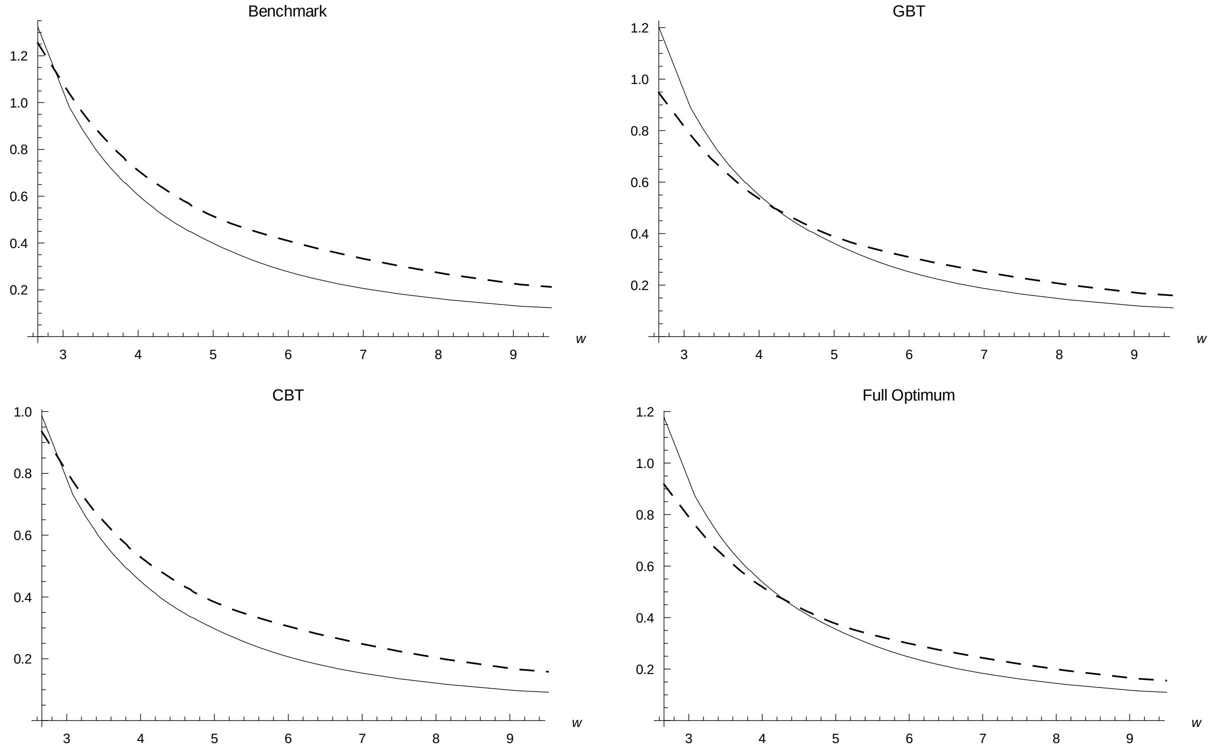

Figure 5: Household Labor Married (solid line = males, dashed = females)

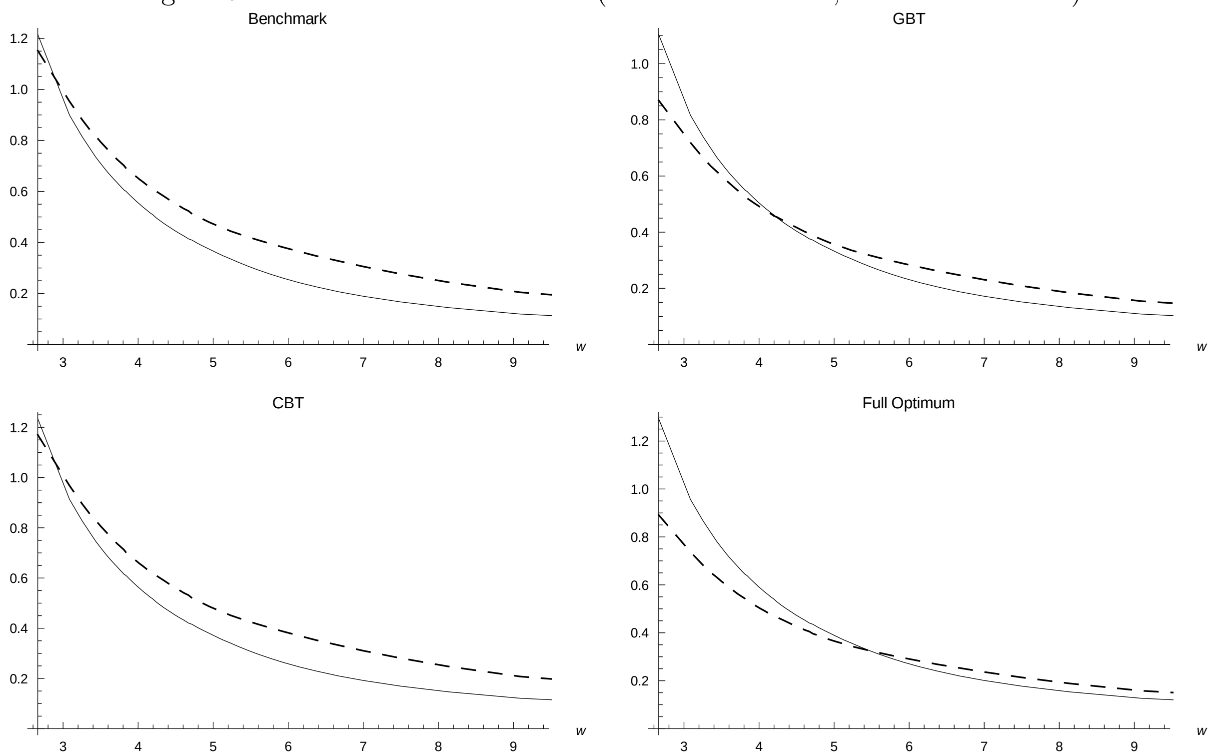

Figure 6: Household Labor Singles (solid line = males, dashed = females)

B Derivations for the model without income effects

First Order Conditions

For expositional simplicity I drop here everywhere the subscript 'i'. To solve problem (6), denote by $\Omega_m = \gamma(U_m - U_m^s)^{\gamma-1}(U_f - U_f^s)^{1-\gamma}$ and $\Omega_f = (1-\gamma)(U_m - U_m^s)^{\gamma}(U_f - U_f^s)^{-\gamma}$ and attach the Lagrange multiplier μ to the government budget constraint. The first order conditions can (after some minor algebra) be written:

$$
\begin{aligned}
c_m: &\quad \Omega_m = \mu \\
c_f: &\quad \Omega_f = \mu \\
\ell_m: &\quad (\ell_m + h_m)^{\phi} = z(1-\tau_m)w_m \\
\ell_f: &\quad (\ell_f + h_f)^{\phi} = z(1-\tau_f)w_f \\
h_m: &\quad (\ell_m + h_m)^{\phi} = 2A\omega_m h_m^{\alpha^c - 1} \\
h_f: &\quad (\ell_f + h_f)^{\phi} = 2A\omega_f h_f^{\alpha^c - 1}
\end{aligned}
$$

and the pooled budget constraint

$$c_m = z((1-\tau_m)w_m\ell_m + (1-\tau_f)w_f\ell_f + G_m + G_f) - c_f$$

Combining the FOC:s for h_m, h_f, ℓ_m and ℓ_f one finds

$$h_j = \left(\frac{2A\omega_j}{z(1-\tau_j)w_j}\right)^{\frac{1}{1-\alpha^c}}, \quad j = m, f$$

Combining the conditions for c_m and c_f yields $\Omega_f = \Omega_m$ and using (4) yields

$$
\begin{aligned}
\gamma(U_f - U_f^s) &= (1-\gamma)(U_m - U_m^s) \\
\gamma(U_f + U_m) &= U_m - (1-\gamma)U_m^s + \gamma U_f^s \\
\gamma\left(c_f + c_m + 2H - \frac{1}{1+\phi}((\ell_f + h_f)^{1+\phi} + (\ell_m + h_m)^{1+\phi})\right) &= U_m - (1-\gamma)U_m^s + \gamma U_f^s \\
\gamma\left(zy + 2H - \frac{1}{1+\phi}((\ell_f + h_f)^{1+\phi} + (\ell_m + h_m)^{1+\phi})\right) &= U_m - (1-\gamma)U_m^s + \gamma U_f^s
\end{aligned}
$$

where total household income is $y = y_m + y_f$ and $y_j = (1-\tau_j)w_j\ell_j + G_j$, $j = m.f$. After some algebra the above can be rewritten as

$$U_m = \gamma\Big(zy + 2H - \frac{1}{1+\phi}((\ell_f + h_f)^{1+\phi} + (\ell_m + h_m)^{1+\phi})\Big) + (1-\gamma)U_m^s - \gamma U_f^s \qquad (32)$$

and by symmetry

$$U_f = (1-\gamma)\Big(zy + 2H - \frac{1}{1+\phi}((\ell_f + h_f)^{1+\phi} + (\ell_m + h_m)^{1+\phi})\Big) \\ + \gamma U_f^s - (1-\gamma)U_m^s \quad (33)$$

Using the expression for Q_i, these expressions can be re-arranged to form (7) and (8).

Effects on intra-household utility

In table 7 the partial derivatives of the effects of taxes on the utility of individual members of a couple are presented. These have been obtained by taking the derivative of expressions (32) and (33) with respect to the tax instruments. Formulas are presented for (i) the uniform linear income tax regime, (ii) gender-based taxation, (iii) couple-based taxation, and (iv) the full optimum.

C Derivation of the elasticity of labor supply

To derive the elasticity I proceed as follows. The FOC for ℓ implies

$$(\ell_m + h_m)^\phi = (c_m + H)^{-\beta} z(1-\tau_m)w_m$$

differentiation wrt to w_m yields

$$\frac{\partial \ell_m}{\partial w_m} + \frac{\partial h_m}{\partial w_m} = -\frac{\beta}{\phi}(c_m + H)^{-\frac{\beta}{\phi}-1}(z(1-\tau_m))^{\frac{1}{\phi}} w_m^{\frac{1}{\phi}} \left[\frac{\partial c_m}{\partial w_m} + \frac{\partial H}{\partial w_m}\right] \\ + (c_m + H)^{-\frac{\beta}{\phi}}(z(1-\tau_m))^{\frac{1}{\phi}} w_m^{\frac{1}{\phi}-1}\frac{1}{\phi}$$

using the FOC for ℓ again:

$$\frac{\partial \ell_m}{\partial w_m} + \frac{\partial h_m}{\partial w_m} = -\frac{\beta}{\phi}\frac{1}{(c_m + H)}(\ell_m + h_m)\left[\frac{\partial c_m}{\partial w_m} + \frac{\partial H}{\partial w_m}\right] + \frac{1}{\phi}\frac{1}{w_m}(\ell_m + h_m)$$

From the expression for h_m I obtain $\frac{\partial h_m}{\partial w_m} = -\frac{1}{1-\alpha^c}\frac{h_m}{w_m}$ hence

$$\frac{\partial \ell_m}{\partial w_m} = -\frac{\beta}{\phi}\frac{1}{(c_m + H)}(\ell_m + h_m)\left[\frac{\partial c_m}{\partial w_m} + \frac{\partial H}{\partial w_m}\right] \\ + \frac{1}{\phi}\frac{1}{w_m}(\ell_m + h_m) + \frac{1}{1-\alpha^c}\frac{h_m}{w_m} \quad (34)$$

Taking the total derivative of the utility function U_m with respect to w_m and setting it to zero is equivalent to

$$\left[\frac{\partial c_m}{\partial w_m} + \frac{\partial H}{\partial w_m}\right] = -\frac{(\ell_m + h_m)^\phi}{(c_m + H)^{-\beta}}\left[\frac{\partial \ell_m}{\partial w_m} + \frac{\partial h_m}{\partial w_m}\right]$$

Table 7: Effects of taxes on intra-household utility

Uniform Linear Tax	Gender-Based Taxation	Couple-Based Taxation	Full Optimum
$\frac{\partial V_m}{\partial \tau} = -\gamma z(y_m + y_f) - (1-\gamma)y_m^s + \gamma y_f^s$	$\frac{\partial V_m}{\partial \tau_m} = -(\gamma z y_m + (1-\gamma)y_m^s)$	$\frac{\partial V_m}{\partial \tau_c} = -\gamma z(y_m + y_f)$	$\frac{\partial V_m}{\partial \tau_m} = -\gamma z y_m < 0$
$\frac{\partial V_m}{\partial G} = \gamma z + (1-2\gamma)$	$\frac{\partial V_m}{\partial G_m} = \gamma z + (1-\gamma) > 0$	$\frac{\partial V_m}{\partial G_c} = \gamma z$	$\frac{\partial V_m}{\partial \tau_m^s} = -(1-\gamma)y_m^s < 0$
$\frac{\partial V_f}{\partial \tau} = -(1-\gamma)z(y_m + y_f) + (1-\gamma)y_m^s - \gamma y_f^s$	$\frac{\partial V_m}{\partial \tau_f} = -\gamma(z y_f - y_f^s)$	$\frac{\partial V_f}{\partial \tau_c} = -(1-\gamma)z(y_m + y_f)$	$\frac{\partial V_m}{\partial G_m} = \gamma z > 0$
$\frac{\partial V_f}{\partial G} = (1-\gamma)z + (-1+2\gamma)$	$\frac{\partial V_m}{\partial G_f} = \gamma(z-1) > 0$	$\frac{\partial V_f}{\partial G_c} = (1-\gamma)z$	$\frac{\partial V_m}{\partial G_m^s} = (1-\gamma) > 0$
			$\frac{\partial V_m}{\partial \tau_f} = -\gamma z y_f < 0$
	$\frac{\partial V_f}{\partial \tau_m} = -((1-\gamma)z y_m - (1-\gamma)y_m^s)$	$\frac{\partial V_m}{\partial \tau_s} = -(1-\gamma)y_m^s + \gamma y_f^s$	$\frac{\partial V_m}{\partial \tau_f^s} = \gamma y_f^s > 0$
	$\frac{\partial V_f}{\partial G_m} = (1-\gamma)z - (1-\gamma) > 0$	$\frac{\partial V_m}{\partial G_s} = (1-2\gamma)$	$\frac{\partial V_m}{\partial G_f} = \gamma z > 0$
	$\frac{\partial V_f}{\partial \tau_f} = -((1-\gamma)z y_f + \gamma y_f^s)$	$\frac{\partial V_f}{\partial \tau_s} = (1-\gamma)y_m^s - \gamma y_f^s$	$\frac{\partial V_m}{\partial G_f^s} = -\gamma < 0$
	$\frac{\partial V_f}{\partial G_f} = (1-\gamma)z + \gamma > 0$	$\frac{\partial V_f}{\partial G_s} = (-1+2\gamma)$	
			$\frac{\partial V_f}{\partial \tau_m} = -(1-\gamma)z y_m < 0$
	$\frac{\partial V_m}{\partial \tau_c} = -\gamma z(y_m + y_f)$		$\frac{\partial V_f}{\partial \tau_m^s} = (1-\gamma)y_m^s > 0$
	$\frac{\partial V_m}{\partial G_c} = \gamma z$		$\frac{\partial V_f}{\partial G_m} = (1-\gamma)z > 0$
	$\frac{\partial V_f}{\partial \tau_c} = -(1-\gamma)z(y_m + y_f)$		$\frac{\partial V_f}{\partial G_m^s} = -(1-\gamma) < 0$
	$\frac{\partial V_f}{\partial G_c} = (1-\gamma)z$		$\frac{\partial V_f}{\partial \tau_f} = -(1-\gamma)z y_f < 0$
			$\frac{\partial V_f}{\partial \tau_f^s} = -\gamma y_f^s < 0$
			$\frac{\partial V_f}{\partial G_f} = (1-\gamma)z > 0$
			$\frac{\partial V_f}{\partial G_f^s} = \gamma > 0$

Hence using the expression for the equilibrium MRS

$$\left[\frac{\partial c_m}{\partial w_m} + \frac{\partial H}{\partial w_m}\right] = z(1-\tau_m)w_m \left[\frac{\partial \ell_m}{\partial w_m} + \frac{\partial h_m}{\partial w_m}\right] \tag{35}$$

it is possible to insert (35) into (34) to obtain

$$\frac{\partial \ell_m}{\partial w_m} = -\frac{\beta}{\phi}\frac{z(1-\tau_m)w_m(\ell_m+h_m)}{(c_m+H)}\left[\frac{\partial \ell_m}{\partial w_m} - \frac{1}{1-\alpha^c}\frac{h_m}{\ell_m}\right] + \frac{1}{\phi}\frac{1}{w_m}(\ell_m+h_m) + \frac{1}{1-\alpha^c}\frac{h_m}{w_m}$$

Define $\zeta \equiv \frac{z(1-\tau_m)w_m(\ell_m+h_m)}{(c_m+H)}$ which is the equilibrium value (to the household) of the total labor supply of the husband divided by the market value of consumption.

$$\frac{\partial \ell_m}{\partial w_m}\left[1+\frac{\beta}{\phi}\zeta\right] = \frac{\beta}{\phi}\zeta\frac{1}{1-\alpha^c}\frac{h_m}{\ell_m} + \frac{1}{\phi}\frac{1}{w_m}(\ell_m+h_m) + \frac{1}{1-\alpha^c}\frac{h_m}{w_m}$$

hence

$$\frac{\partial \ell_m}{\partial w_m}\left[1+\frac{\beta}{\phi}\zeta\right] = \frac{1}{1-\alpha^c}\frac{h_m}{w_m}\left[1+\frac{\beta}{\phi}\zeta\right] + \frac{1}{\phi}\frac{1}{w_m}(\ell_m+h_m)$$

multiply by ϕ

$$\frac{\partial \ell_m}{\partial w_m}\left[\beta\zeta+\phi\right] = \frac{1}{1-\alpha^c}\frac{h_m}{w_m}\left[\beta\zeta+\phi\right] + \frac{1}{w_m}(\ell_m+h_m)$$

using the definition of the elasticity

$$\epsilon\left[\beta\zeta+\phi\right] = \frac{1}{1-\alpha^c}\frac{h_m}{\ell_m}\left[\beta\zeta+\phi\right] + (1+\frac{h_m}{\ell_m})$$

or

$$\epsilon = \frac{1}{1-\alpha^c}\frac{h_m}{\ell_m} + \frac{1}{\phi+\beta\zeta}\left(1+\frac{h_m}{\ell_m}\right)$$

Finally this can be written as

$$\epsilon = \frac{1}{\phi+\beta\zeta} + \left(\frac{1}{\phi+\beta\zeta} + \frac{1}{1-\alpha^c}\right)\frac{h_m}{\ell_m}$$

D Derivation of linear income tax formulas

The formulas derived below follow from the maximization of (9) subject to (10) under various assumptions regarding the tax instruments available to the government.

Uniform linear income tax

Define $\Phi^c_m = \gamma z, \Phi^c_f = (1-\gamma)z$ and $\Phi^s_j = 1$. It is useful to expand expression (12) using table 7:

$$G : \sum_{ij} \pi_{ij} \left(\eta W'(V^s_{ij}) \frac{\partial V^s_{ij}}{\partial G} + (1-\eta) W'(V^c_{ij}) \frac{\partial V^c_{ij}}{\partial G} \right) = \lambda$$

$$\Longleftrightarrow \sum_i \pi_i \left(\frac{\eta}{2} \sum_{j=m,f} \frac{W'(V^s_{ij})}{\lambda} \right.$$
$$\left. + \frac{(1-\eta)}{2} \left[\sum_{j=m,f} \frac{W'(V^c_{ij})\Phi^c_j}{\lambda} + \frac{W'(V^c_{im}) - W'(V^c_{if})}{\lambda} \left(\frac{1}{2} - \gamma \right) \right] \right) = 1$$

and similarly, using expression (11)

$$\tau : \sum_{ij} \pi_{ij} \left(\eta W'(V^s_{ij}) \frac{\partial V^s_{ij}}{\partial \tau} + (1-\eta) W'(V^c_{ij}) \frac{\partial V^c_{ij}}{\partial \tau} \right) = \eta \sum_{ijk} -\pi^k_{ij} \lambda \left(y^k_{ij} + \tau \frac{\partial y^k_{ij}}{\partial \tau} \right)$$

$$\Longleftrightarrow \sum_i \pi_i \left(\frac{\eta}{2} \sum_{j=m,f} \frac{W'(V^s_{ij})}{\lambda} y^s_{ij} + + \frac{(1-\eta)}{2} \left[\sum_{j=m,f} \frac{W'(V_{ij})\Phi^c_{ij}}{\lambda} (y_{im} + y_{if}) \right.\right.$$
$$\left.\left. + \frac{W'(V^c_{im}) - W'(V^c_{if})}{\lambda} \left[(1-\gamma) y^s_{im} - \gamma y^s_{if} \right] \right] \right) = \sum_{ijk} -\pi^k_{ij} \lambda \left(y^k_{ij} + \tau \frac{\partial y^k_{ij}}{\partial \tau} \right) \quad (36)$$

Hence the following proposition is established:

Proposition 7 (Uniform Linear Tax Rate) *The optimal tax rate is*

$$\tau = \frac{1}{\overline{y}'_m} \sum_i \pi_i \Big[\eta \sum_{j=m,f} \left(\vartheta^s_{ij} - 1 \right) y^s_{ij} + (1-\eta) \Big(\left(\vartheta^c_i - 1 \right) (y^c_{im} + y^c_{if}) + \quad (37)$$
$$+ \frac{(1-\eta)}{\eta} \frac{W'(V^c_{im}) - W'(V^c_{if})}{\lambda} \left[(1-\gamma) y^s_{im} - \gamma y^s_{if} \right] \Big) \Big] \quad (38)$$

where $\overline{y}' = \sum_{ijk} \pi^k_{ij} \frac{\partial y^k_{ij}}{\partial \tau}$ *is the average compensated earned income response across the universe of taxpayers.*

Proof of Proposition 2 (Gender Based Taxation)

Transfers

The first order conditions following maximization of (9) are[48]

$$G_m : \sum_i \pi_i \left(\eta W'(V^s_{im}) \frac{\partial V^s_{im}}{\partial G_m} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G_m} \right) = 1$$

$$G_f : \sum_i \pi_i \left(\eta W'(V^s_{if}) \frac{\partial V^s_{if}}{\partial G_f} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G_f} \right) = 1$$

using the expressions in section B and dividing by λ I obtain

$$G_m : \sum_i \pi_i \left(\eta \frac{W'(V^s_{im})}{\lambda} + (1-\eta) \frac{W'(V_{im})[\gamma z + (1-\gamma)] + W'(V_{if})[(1-\gamma)z - (1-\gamma)]}{\lambda} \right) = 1$$

$$G_f : \sum_i \pi_i \left(\eta \frac{W'(V^s_{if})}{\lambda} + (1-\eta) \frac{W'(V_{im})[\gamma(z-1)] + W'(V_{if})[(1-\gamma)z + \gamma]}{\lambda} \right) = 1.$$

This can be written as

$$G_m : \sum_i \pi_i \left(\eta \vartheta^s_{im} + (1-\eta)\vartheta^c_i + (1-\eta)(1-\gamma) \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = 1 \qquad (39)$$

$$G_f : \sum_i \pi_i \left(\eta \vartheta^s_{if} + (1-\eta)\vartheta^c_i - (1-\eta)\gamma \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = 1 \qquad (40)$$

where $\vartheta^c_i = \vartheta^c_{im} + \vartheta^c_{if} = z \left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma) \frac{W'(V_{if})}{\lambda} \right)$ and $\vartheta^s_{ij} = \frac{W'(V^s_{ij})}{\lambda}$.

[48]Note that half of all agents are men and half are women. The fraction of singles and couples are η and $1-\eta$ respectively. In their original form the formulas below contain $\frac{1}{2}$ in every term, but they have here been pre-multiplied by 2 for expositional clarity.

Marginal tax rates

The first order conditions are

$$\tau_m : \sum_i \pi_i \left(\eta W'(V^s_{im}) \frac{\partial V^s_{im}}{\partial \tau_m} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_m} \right)$$
$$= - \sum_i \pi_i \lambda \left(\eta (y^s_{im} + \tau_m \frac{\partial y^s_{im}}{\partial \tau_m}) + (1-\eta)(y_{im} + \tau_m \frac{\partial y_{im}}{\partial \tau_m}) \right)$$
$$\tau_f : \sum_i \pi_i \left(\eta W'(V^s_{if}) \frac{\partial V^s_{if}}{\partial \tau_f} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_f} \right)$$
$$= - \sum_i \pi_i \lambda \left(\eta (y^s_{if} + \tau_f \frac{\partial y^s_{if}}{\partial \tau_f}) + (1-\eta)(y_{if} + \tau_f \frac{\partial y_{if}}{\partial \tau_f}) \right)$$

which can be simplified to

$$\tau_m : \sum_i \pi_i (\frac{\eta}{\lambda} W'(V^s_{im}) y^s_{im} +$$
$$(1-\eta) \frac{[W'(V_{im})\gamma z + W'(V_{if})(1-\gamma)z] y_{im} + (1-\gamma)[W'(V_{im}) - W'(V_{if})] y^s_{im}}{\lambda})$$
$$= \sum_i \pi_i (\eta (y^s_{im} + \tau_m \frac{\partial y^s_{im}}{\partial \tau_m}) + (1-\eta)(y_{im} + \tau_m \frac{\partial y_{im}}{\partial \tau_m}))$$
$$\tau_f : \sum_i \pi_i (\frac{\eta}{\lambda} W'(V^s_{if}) y^s_{if} +$$
$$(1-\eta) \frac{[W'(V_{im})\gamma z + W'(V_{if})(1-\gamma)z] y_{if} - \gamma [W'(V_{im}) - W'(V_{if}))] y^s_{if}}{\lambda})$$
$$= \sum_i \pi_i (\eta (y^s_{if} + \tau_f \frac{\partial y^s_{if}}{\partial \tau_m}) + (1-\eta)(y_{if} + \tau_f \frac{\partial y_{if}}{\partial \tau_f}))$$

Hence it is possible to write

$$\tau_m = \frac{1}{\overline{y}'} \sum_i \Big[\eta \left(\frac{W'(V^s_{im})}{\lambda} - 1 \right) y^s_{im} +$$
$$(1-\eta) \left(\sum_{j=m,f} \frac{W'(V^c_{ij}) \Phi^c_{ij}}{\lambda} - 1 \right) y^c_{im} + (1-\eta)(1-\gamma) \left(\frac{W'(V_{im}) - W'(V_{if}))}{\lambda} \right) y^s_{im} \Big]$$

and

$$\tau_f = \frac{1}{\overline{y}'} \sum_i \Big[\eta \left(\frac{W'(V^s_{if})}{\lambda} - 1 \right) y^s_{if} +$$
$$(1-\eta) \left(\sum_{j=m,f} \frac{W'(V^c_{ij}) \Phi^c_{ij}}{\lambda} - 1 \right) y^c_{if} - (1-\eta)\gamma \left(\frac{W'(V_{im}) - W'(V_{if}))}{\lambda} \right) y^s_{if} \Big]$$

Proof of Proposition 4 (Couple-Based Tax)

Using expression (12) and table 7 I obtain

$$G^s : \sum_i \pi_i \left(\frac{1}{2} \sum_{j=m,f} \left[\eta W'(V_{ij}^s) \frac{\partial V_{ij}^s}{\partial G_s} + (1-\eta) W'(V_{ij}^c) \frac{\partial V_{ij}^c}{\partial G_s} \right] \right) = \eta\lambda \Longleftrightarrow$$

$$\Longleftrightarrow \sum_i \pi_i \left(\sum_{j=m,f} \frac{1}{2} \frac{W'(V_{ij}^s)}{\lambda} + \frac{(1-\eta)}{\eta} \left[\frac{W'(V_{im}^c)}{\lambda} \left(\frac{1}{2} - \gamma \right) + \frac{W'(V_{if}^c)}{\lambda} \left(-\frac{1}{2} + \gamma \right) \right] \right) = 1$$

$$\Longleftrightarrow \sum_i \pi_i \left(\sum_{j=m,f} \frac{1}{2} \frac{W'(V_{ij}^s)}{\lambda} + \frac{(1-\eta)}{\eta} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left(\frac{1}{2} - \gamma \right) \right) = 1$$

If $\gamma = 1/2$ this can be written:

$$E[\vartheta^s] = 1 \tag{41}$$

where the expectation is taken over i and j. The corresponding condition for G^c is:

$$G^c : \sum_i \pi_i \left(\frac{1}{2} \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G_c} \right) = \lambda \Longleftrightarrow \sum_i \pi_i \left(\frac{1}{2} \sum_{j=m,f} \frac{W'(V_{ij}) \Phi_j^c}{\lambda} \right) = 1$$

and (irrespective of γ) the above expression can be written:

$$E[\vartheta^c] = 1$$

To derive an expression for τ^s, expression (37) and table 7 gives:

$$\tau^s \quad : \quad \sum_{ij} \pi_{ij} \left(\eta W'(V_{ij}^s) \frac{\partial V_{ij}^s}{\partial \tau^s} + (1-\eta) W'(V_{ij}^c) \frac{\partial V_{ij}^c}{\partial \tau^s} \right) \quad = \quad \eta \sum_{ij} -\pi_{ij} \lambda \left(y_{ij}^s + \tau^s \frac{\partial y_{ij}^s}{\partial \tau^s} \right)$$

$$\Longleftrightarrow \sum_i \pi_i \Bigg(\frac{1}{2} \sum_{j=m,f} \frac{W'(V_{ij}^s)}{\lambda} y_{ij}^s +$$
$$\frac{(1-\eta)}{\eta} \frac{1}{2} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left[(1-\gamma) y_{im}^s - \gamma y_{if}^s \right] \Bigg) = \sum_i \pi_i \sum_{j=m,f} \left(y_{ij}^s + \tau^s \frac{\partial y_{ij}^s}{\partial \tau^s} \right)$$

$$\Longleftrightarrow \sum_i \pi_i \Bigg(\sum_{j=m,f} \left(\frac{W'(V_{ij}^s)}{\lambda} - 1 \right) y_{ij}^s +$$
$$\frac{(1-\eta)}{\eta} \frac{1}{2} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left[(1-\gamma) y_{im}^s - \gamma y_{if}^s \right] \Bigg) = \tau^s \sum_{ij} \pi_{ij} \frac{\partial y_{ij}^s}{\partial \tau^s}$$

A re-arrangement of terms gives:

$$\tau^s = \frac{1}{\overline{y}^{s'}} \sum_i \pi_i \frac{1}{2} \Big[(\vartheta_{im}^s - 1) y_{im}^s + (\vartheta_{if}^s - 1) y_{if}^s +$$
$$\frac{(1-\eta)}{\eta} \frac{W'(V_{im}^c) - W'(V_{if}^c)}{\lambda} \left[(1-\gamma) y_{im}^s - \gamma y_{if}^s \right] \Big]$$

The same procedure for τ^c yields

$$\tau^c : \sum_{ij} \pi_{ij} \left(W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau^c} \right) = \sum_{ij} -\pi_{ij} \lambda \left(y_{ij} + \tau^c \frac{\partial y_{ij}}{\partial \tau^c} \right)$$

or

$$\tau^c : \sum_{ij} \pi_{ij} \left(W'(V_{ij}) \Phi^c_{ij} (y_{im} + y_{if}) \right) = \sum_{ij} \pi_{ij} \lambda \left(y_{ij} + \tau^c \frac{\partial y_{ij}}{\partial \tau^c} \right)$$

and

$$\sum_i \pi_i \left(\sum_{j=m,f} \frac{W'(V_{ij}) \Phi^c_{ij}}{\lambda} - 1 \right) (y_{im} + y_{if}) = \tau_c \sum_i \pi_i \sum_{j=m,f} \frac{\partial y_{ij}}{\partial \tau^c} \tag{42}$$

or

$$\tau^c = \frac{\mathbf{cov}(\vartheta^c, y_m + y_f)}{\overline{y^c}'}$$

Proof of Proposition 6 (Full Optimum, Transfers)

$$G^s_m : \sum_i \pi_i \left(\eta W'(V^s_{im}) \frac{\partial V^s_{im}}{\partial G^s_m} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G^s_m} \right) = \eta \lambda$$

$$G^s_f : \sum_i \pi_i \left(\eta W'(V^s_{if}) \frac{\partial V^s_{if}}{\partial G^s_f} + (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G^s_f} \right) = \eta \lambda$$

$$G_m : \sum_i \pi_i \left(\sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G_m} \right) = \lambda$$

$$G_f : \sum_i \pi_i \left(\sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial G_f} \right) = \lambda$$

or

$$G^s_m : \sum_i \pi_i \left(\eta \frac{W'(V^s_{im})}{\lambda} + (1-\eta)(1-\gamma) \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = \eta$$

$$G^s_f : \sum_i \pi_i \left(\eta \frac{W'(V^s_{if})}{\lambda} - (1-\eta)\gamma \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) = \eta$$

$$G_m : \sum_i \pi_i z \left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma) \frac{W'(V_{if})}{\lambda} \right) = 1$$

$$G_f : \sum_i \pi_i z \left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma) \frac{W'(V_{if})}{\lambda} \right) = 1$$

The last two equations are identical since it doesn't matter who in the family receives the transfer. There is no need to differentiate lump-sum transfers within the couple, hence I can set $G_m = G_f = G$.

Proof of Proposition 5 (Full Optimum, Tax Rates)

$$\tau_m^s : \sum_i \pi_i \left(\eta W'(V_{im}^s) \frac{\partial V_{im}^s}{\partial \tau_m^s} + \right.$$

$$\left. (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_m^s} \right) = -\eta \sum_i \pi_i \lambda (y_{im}^s + \tau_m \frac{\partial y_{im}^s}{\partial \tau_m^s})$$

$$\tau_f^s : \sum_i \pi_i \left(\eta W'(V_{if}^s) \frac{\partial V_{if}^s}{\partial \tau_f^s} + \right.$$

$$\left. (1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_f^s} \right) = -\eta \sum_i \pi_i \lambda (y_{if}^s + \tau_f \frac{\partial y_{if}^s}{\partial \tau_f^s})$$

$$\tau_m : \sum_i \pi_i \left((1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_m} \right) = -(1-\eta) \sum_i \pi_i \lambda (y_{im} + \tau_m \frac{\partial y_{im}}{\partial \tau_m})$$

$$\tau_f : \sum_i \pi_i \left((1-\eta) \sum_{j=m,f} W'(V_{ij}) \frac{\partial V_{ij}}{\partial \tau_f} \right) = -(1-\eta) \sum_i \pi_i \lambda (y_{if} + \tau_f \frac{\partial y_{if}}{\partial \tau_f})$$

or

$$\tau_m^s : \sum_i \pi_i \left(\eta \frac{W'(V_{im}^s)}{\lambda} + \right.$$

$$\left. (1-\eta)(1-\gamma) \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) y_{im}^s = \eta \sum_i \pi_i (y_{im}^s + \tau_m \frac{\partial y_{im}^s}{\partial \tau_m^s})$$

$$\tau_f^s : \sum_i \pi_i \left(\eta \frac{W'(V_{if}^s)}{\lambda} - \right.$$

$$\left. (1-\eta)\gamma \frac{W'(V_{im}) - W'(V_{if})}{\lambda} \right) y_{if}^s = \eta \sum_i \pi_i (y_{if}^s + \tau_f \frac{\partial y_{if}^s}{\partial \tau_f^s})$$

$$\tau_m : \sum_i \pi_i z \left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma) \frac{W'(V_{if})}{\lambda} \right) y_{im} = \sum_i \pi_i (y_{im} + \tau_m \frac{\partial y_{im}}{\partial \tau_m})$$

$$\tau_f : \sum_i \pi_i z \left(\gamma \frac{W'(V_{im})}{\lambda} + (1-\gamma) \frac{W'(V_{if})}{\lambda} \right) y_{if} = \sum_i \pi_i (y_{if} + \tau_f \frac{\partial y_{if}}{\partial \tau_f})$$

solving for the tax rates gives the desired result.

First order conditions, model with income effects

I assume both spouses are sufficiently productive in both sectors so that the optimum is interior. The first order conditions are

$$
\begin{aligned}
c_m: &\quad \Omega_m(c_m + H)^{-\beta} = \mu \\
c_f: &\quad \Omega_f(c_f + H)^{-\beta} = \mu \\
\ell_m: &\quad \Omega_m(\ell_m + h_m)^{\phi} = \mu z(1-\tau_m)w_m \\
\ell_f: &\quad \Omega_f(\ell_f + h_f)^{\phi} = \mu z(1-\tau_f)w_f \\
h_m: &\quad \Omega_m(\ell_m + h_m)^{\phi} = \left[\Omega_m(c_m + H)^{-\beta} + \Omega_f(c_f + H)^{-\beta}\right]\omega_m h_m^{\alpha^c - 1} \\
h_f: &\quad \Omega_f(\ell_f + h_f)^{\phi} = \left[\Omega_m(c_m + H)^{-\beta} + \Omega_f(c_f + H)^{-\beta}\right]\omega_f h_f^{\alpha^c - 1}
\end{aligned}
$$

and

$$c_m = z((1-\tau_m)w_m\ell_m + (1-\tau_f)w_f\ell_f + G_m + G_f) - c_f$$

E Household economies of scale

Here I describe how the economies of scale parameters are determined. In the model analyzed there are economies of scale in market consumption and household consumption. For general production technologies, economies of scale are not described by a 'single parameter', but are instead endogenously determined and depends on the process by which the household transforms income into market consumption and household work into household consumption. Following Lewbel and Pendakur (2008) I assume an individual needs 2/3 of the resources available to a couple, as single, in order to reach the same utility he or she would enjoy when living as a couple.[49] To be able to make progress I need to make some simplifying assumptions. Suppose an individual receives $\frac{1}{2}$ of the total consumption of the household when living in a couple. Furthermore assume individuals have the same market productivity and labor supply irrespective of gender and marital status. As before, I assume no gender differences in the marginal productivity of household work. I wish to approximate a situation where individuals reach the same indifference curve when they are single and when they are married given that resources

[49]This is an approximation because the indifference scale depends on the degree of publicness of goods, production technologies and the sharing rule. To obtain a more exact figure would require structural estimation of the current model, an exercise which lies out of the scope of the present paper.

when single are 'scaled up' so that they amount to $\frac{2}{3}$ of the resources of the couple. One way to approximate this is to require that a single individual would need a market income of $y_j^s = \frac{2}{3}y^c$ when the total resources of the couple is y^c to reach the same market consumption he or she would enjoy in the couple. In other words, it would be required that $c_j^s = c_j$ when $y_j^s = \frac{2}{3}y^c$. This yields $\frac{2}{3}y^c = z\frac{1}{2}y^c$, since when married, each household member gets $\frac{y^c}{2}$ but generates utility more efficiently since $z > 1$. The solution to this gives $z = \frac{4}{3} = 1.33$. Because of decreasing-returns-to-scale in household production agents work in the household until their marginal product equals the fixed price of the market good. With the price and economies of scale of the market good fixed, the parameters in the household production technology determine the ratio of household consumption to market goods. I calibrate the parameters of the production function to result in empirically relevant ratios of h_j/ℓ_j and h_j^s/ℓ_j^s, $j = m, f$ as well as the ratio H_j^s/H.

WORKING PAPERS
Uppsala Center for Fiscal Studies
Editor: Håkan Selin

2009:1 Sören Blomquist and Håkan Selin, Hourly Wage Rate and Taxable Labor Income Responsiveness to Changes in Marginal Tax Rates. 31 pp.

2009:2 Luca Micheletto, Optimal nonlinear redistributive taxation and public good provision in an economy with Veblen effects. 26 pp.

2009:3 Håkan Selin, The Rise in Female Employment and the Role of Tax Incentives. An Empirical Analysis of the Swedish Individual Tax Reform of 1971. 38 pp.

2009:4 Håkan Selin, Marginal tax rates and tax-favoured pension savings of the self-employed Evidence from Sweden. 32 pp.

2009:5 Tobias Lindhe and Jan Södersten, Dividend taxation, share repurchases and the equity trap. 27 pp.

2009:6 Che-Yan Liang, Nonparametric Structural Estimation of Labor Supply in the Presence of Censoring. 48 pp.

2009:7 Sören Blomquist, Vidar Christiansen and Luca Micheletto, Public Provision of Private Goods and Nondistortionary Marginal Tax Rates: Some further Results. 42 pp.

2009:8 Laurent Simula and Alain Trannoy, Optimal Income Tax under the Threat of Migration by Top-Income Earners. 26 pp.

2009:9 Laurent Simula and Alain Trannoy, Shall We Keep Highly Skilled at Home? The Optimal Income Tax Perspective. 26 pp.

2009:10 Michael Neugart and Henry Ohlsson, Economic incentives and the timing of births: Evidence from the German parental benefit reform 2007, 21 pp.

2009:11 Laurent Simula, Optimal Nonlinear Income Tax and Nonlinear Pricing: Optimality Conditions and Comparative Static Properties, 25 pp.

2009:12 Ali Sina Onder and Herwig Schlunk, Elderly Migration, State Taxes, and What They Reveal, 26 pp.

2009:13 Ohlsson, Henry, The legacy of the Swedish gift and inheritance tax, 1884-2004, 26 pp.

2009:14 Onder, Ali Sina, Capital Tax Competition When Monetary Competition is Present, 29 pp.

2010:1 Sören Blomquist and Laurent Simula, Marginal Deadweight Loss when the Income Tax is Nonlinear. 21 pp.

2010:2 Marcus Eliason and Henry Ohlsson, Timing of death and the repeal of the Swedish inheritance tax. 29 pp.

2010:3 Mikael Elinder, Oscar Erixson and Henry Ohlsson, The Effect of Inheritance Receipt on Labor and Capital Income: Evidence from Swedish Panel Data. 28 pp.

2010:4 Jan Södersten and Tobias Lindhe, The Norwegian Shareholder Tax Reconsidered. 21 pp.

2010:5 Anna Persson and Ulrika Vikman, Dynamic effects of mandatory activation of welfare participants. 37 pp.

2010:6 Ulrika Vikman, Does Providing Childcare to Unemployed Affect Unemployment Duration? 43 pp.

2010:7 Per Engström, Bling Bling Taxation and the Fiscal Virtues of Hip Hop. 12 pp.

2010:8 Niclas Berggren and Mikael Elinder, Is tolerance good or bad for growth? 34 pp.

2010:9 Heléne Lundqvist, Matz Dahlberg and Eva Mörk, Stimulating Local Public Employment: Do General Grants Work? 35pp.

2010:10 Marcus Jacob and Martin Jacob, Taxation, Dividends, and Share Repurchases: Taking Evidence Global. 42 pp.

2010:11 Thomas Aronsson and Sören Blomquist, The Standard Deviation of Life-Length, Retirement Incentives, and Optimal Pension Design. 24 pp.

2010:12 Spencer Bastani, Sören Blomquist and Luca Micheletto, The Welfare Gains of Age Related Optimal Income Taxation. 48 pp.

2010:13 Adrian Adermon and Che-Yuan Liang, Piracy, Music, and Movies: A Natural Experiment. 23 pp.

2010:14 Spencer Bastani, Sören Blomquist and Luca Micheletto, Public Provision of Private Goods, Tagging and Optimal Income Taxation with Heterogeneity in Needs. 47 pp.

2010:15 Mattias Nordin, Do voters vote in line with their policy preferences? The role of information. 33 pp.

2011:1 Matz Dahlberg, Karin Edmark and Heléne Lundqvist, Ethnic Diversity and Preferences for Redistribution. 43 pp.

2011:2 Tax Policy and Employment: How Does the Swedish System Fare? Jukka Pirttilä and Håkan Selin. 73 pp.

2011:3 Alessandra Casarico, Luca Micheletto and Alessandro Sommacal, Intergenerational transmission of skills during childhood and optimal public policy. 48 pp.

2011:4 Jesper Roine and Daniel Waldenström, On the Role of Capital Gains in Swedish Income Inequality. 32 pp.

2011:5 Matz Dahlberg, Eva Mörk and Pilar Sorribas-Navarro, Do Politicians' Preferences Matter for Voters' Voting Decisions?

2011:6 Sören Blomquist, Vidar Christiansen and Luca Micheletto, Public provision of private goods, self-selection and income tax avoidance. 27 pp.

2011:7 Bas Jacobs, The Marginal Cost of Public Funds is One. 36 pp.

2011:8 Håkan Selin, What happens to the husband's retirement decision when the wife's retirement incentives change? 37 pp.

2011:9 Martin Jacob, Tax Regimes and Capital Gains Realizations. 42 pp.

2011:10 Katarina Nordblom and Jovan Zamac, Endogenous Norm Formation Over the Life Cycle – The Case of Tax Evasion. 30 pp.

2011:11 Per Engström, Katarina Nordblom, Henry Ohlsson and Annika Persson, Loss evasion and tax aversion. 43 pp.

2011:12 Spencer Bastani and Håkan Selin, Bunching and Non-Bunching at Kink Points of the Swedish Tax Schedule. 39 pp.

2011:13 Svetlana Pashchenko and Ponpoje Porapakkarm, Welfare costs of reclassification risk in the health insurance market. 41 pp.

2011:14 Hans A Holter, Accounting for Cross-Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure. 56 pp.

2012:1 Stefan Hochguertel and Henry Ohlsson, Who is at the top? Wealth mobility over the life cycle. 52 pp.

2012:2 Karin Edmark, Che-Yuan Liang, Eva Mörk and Håkan Selin, Evaluation of the Swedish earned income tax credit. 39 pp.

2012:3 Martin Jacob and Jan Södersten, Mitigating shareholder taxation in small open economies? 13 pp.

2012:4 Alexander M Gelber, Taxation and the Earnings of Husbands and Wives: Evidence from Sweden. 56 pp.

2012:5 Firouz Gahvari and Luca Micheletto, Monetary policy and redistribution: What can or cannot be neutralized with Mirrleesian taxes. 39 pp.

2012:6 Spencer Bastani, Gender-Based and Couple-Based Taxation. 55 pp.